THE
ECONOMIC PRINCIPLES
OF
ELECTRICAL DISTRIBUTION

SAYERS

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BY

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PREFACE

THIS book may be regarded as a sequel to one published in 1924, under the title of *Electricity Supply Costs and Charges*, but particularly relating to the subject of distribution costs. In the earlier work the more detailed study was devoted to generation costs. The importance of distribution costs was not under-estimated, but material was not abundant and discussion would necessarily have been more theoretical than practical.

Conditions have changed, the subject of distribution costs has become of greater interest, more information is available; the nature of the loads is changing; the more effective utilization of distribution capital has become a problem, at once of greater interest, and more capable of practical solution.

The Report of the McGowan Committee has given statistical information not previously available. The data there presented show a wide "spread" of distribution costs per kilowatt-hour sold by British undertakings all the way from 2·4814d. to 0·5881d., in the six classes of undertakings grouped by the magnitudes of sales; a wide spread in sales per pound of capital employed, and in other relevant data.

Hence a study of first principles seems called for. This book does not claim to go beyond that. Technical subjects are dealt with from that point of view. There is an abundant literature upon the construction of distributing systems to which the author makes no attempt to add. There is just now (September, 1937,) a spate of utterances upon the reorganization of distribution in Britain which involve or express considerations of a quasi-political nature which will presumably be synthetized by legislation. This book affords no guidance upon such matters.

During the gestation of the work, there have been unusual

fluctuations in the prices of materials which seem likely to continue for some time. Such uncertainties, and those of the future course of interest rates upon capital, add to the difficulties of forecasting important data needed for the economic design of distributing systems. These uncertainties impose some degree of caution. But it is always possible to assign limits to the variable elements and to see how the variations affect the "most economical" design.

It is hoped that the work will prove useful to all interested in the subject.

H. M. S.

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CHAPTER I

GENERAL REMARKS

In the commercial business of providing a supply of electrical energy the cost of distribution has become an item of ever-increasing importance. The cost of generation has been reduced at a remarkably high rate. The most modern steam-driven power stations generate one kilowatt-hour for about 1 lb. of coal of average calorific value; over 2 000 kWh. per ton of coal burned. There is still room for improvement in the thermal efficiency of such stations, which now approaches 30 per cent; improvement seems to depend upon the production of steels, etc., able to endure higher temperatures than the maxima now in use, or the more remote possibility of a new prime mover. The higher thermal efficiency of internal combustion engines is not convertible into lower cost of large scale generation, even where natural gas and fuel oil are cheap they are burned under boilers to supply steam turbines.

There is more reason to expect a reduction of average generating costs in the near future from the higher load factor of efficient stations; and the abandonment of the less efficient—a process which in Great Britain is being accelerated by the Central Electricity Board's operations—than from any radical innovation in coal-consuming power plant. That process promises a continuance of the downward trend of generating costs for some time to come.

On the distributing side of the industry there has been an opposite trend, the average cost of delivering a kilowatt-hour to the ultimate consumer has risen, and seems likely to continue to rise. The capital charges involved in distribution are the largest item; the capital expenditure per kilowatt-hour sold has risen of late years. This is not due so much to technical

deficiency or lack of progress in invention or design; on the contrary the means of transmission and distribution have been made more efficient and less costly for a given duty. The chief reasons why the capital costs have risen are : (1) areas of supply have been extended into districts farther from generating stations; (2) the growth and nature of the loads in the original areas of supply have been in excess of the estimates upon which the distributors were planned; (3) consequent upon (2) new mains, sometimes entirely new systems of distribution, have been constructed and the old works scrapped long before the latter were in any sense worn out. So there is a good deal of capital standing for physical assets which have disappeared. That cause should cease to operate within a relatively short time.

To enlarge a little upon the first-named reason; if an area of supply lying wholly within a radius of, say, three miles from a generating station is extended to double that radius, two consequences follow. As the whole of the supply to the new area has to be transmitted to the boundary of the old area and then spread across the radial breadth of the new area, the average distance of transmission to the consumers in the new area will be about doubled. Whilst the new area is three times as extensive as the old one, it is generally less densely populated so that there are fewer consumers per mile of mains with, consequently, a smaller annual consumption in proportion to the capital employed. Where extensions of that kind have been planned, cheaper methods (such as the use of overhead mains instead of underground mains) have to some extent countered the effect of the lesser density of consumption; and it is to be expected that consumption will increase to a reasonably remunerative level. As to the second and third reasons above mentioned—scrapping of distributing plant because the load has outgrown its capacity—it is a standing difficulty in the planning of distributing systems that the estimates of future loading have to be made upon inadequate data. Perhaps one should say have had to be made in the past upon inadequate data; more are available now. Estimates based upon probable lighting demand—the normal basis up to, say, twenty-five years ago—have become entirely inadequate. Additions to the capacities of the older systems have been required long before the economic life of the mains had expired or their cost written down. Reinforcing mains by

additional cables or replacement by larger ones is an expensive business, especially if it involves breaking up the streets. That street work is a serious addition to the cost of the mains themselves; distribution plans should therefore be made elastic so that growth of load may be met by a more economical method than again digging up the streets to lay more cables. Over-estimates of load put excessive capital charges upon distribution until the load grows up to the estimates. Over-estimates have been less common in the past than under-estimates.

Distribution mains have the feature that the capital is immobilized; it is buried; if it is not as fruitful as was expected, it cannot be dug up and planted elsewhere.

With the experience of nearly half a century there is now a body of data for guidance in estimating the loads in a projected system, or extension of one existing. It is by no means complete, conditions are not static, the designer still has to do a good deal of scientific guessing. Twenty-five years ago one did not expect appreciable demands for electric cooking and heating anywhere; it was not safe to reckon on getting a lighting load in more than half the houses rented at about £40 to £70 per annum in suburban London, for example. To-day it would be unsafe to expect less than half of such houses to have an electric cooker or water-heater; perhaps both. It is not yet possible to follow the water engineer who can say pretty accurately that a certain district will consume 30 or 35 gallons per day per head of population; but it is known—where proper records have been kept—what rates of increase may be expected in districts of defined character. Provision for elasticity of growth is called for; as far as possible the provision should not unduly load the income with capital charges in advance of its growth.

In new rural areas data for estimates are scanty. The probability is that the demands of farmers will increase, but at what rate is somewhat problematical. In these areas overhead lines are usually essential; the mechanically safe minimum line structure provides load capacity, unlikely to be exhausted for some years.

In the old-established undertakings the growth of load and extensions of area beyond anticipation compelled the adoption of a hand-to-mouth procedure; the systems have grown in a patchwork fashion; no one designed them as a whole.

This is not a condemnation of the engineers concerned; they

had to deal with changing circumstances beyond their power to forecast. In this country many of the early stations generated continuous current distributed at 100 volts by three-wire systems. The range of economic distribution under these conditions and with a lighting load having a load-factor of the order of 12 per cent was under a mile. In many cases both mains and stations became overloaded rather rapidly; in some additional stations were built, an expensive proceeding where the whole area was a densely built-up one with poor access for fuel and no possibility of condensing. The advance to 200 volts made possible by improvements in lamps for that pressure helped the mains but not the stations. The next step was to put larger stations in more favourable positions, generating three-phase a.c. at 6·6 kV., sometimes more, and to convert the original stations into converting substations still delivering continuous current over three-wire distributors.

By a process which is not yet complete, the ultimate system in these densely loaded areas will pretty certainly become the three-phase four-wire with 400 volts between outers and 230 volts outers to neutral. There are many areas wherein both kinds of distribution are in existence; a transition stage. The successive changes have each added something to capital costs and charges. One consequence is that the distribution capital charges of old-established systems are not trustworthy data for contemporary estimates.

Any new area to be equipped must be studied and planned for as a special case, but there are general principles applicable.

In the following chapters it is intended to outline the study required, and to see how economic principles can be followed in the application of existing material and methods.

CHAPTER II

RUNNING AND FIXED CHARGES

ELEMENTARY analysis of the costs of distribution divides them into two main classes: (1) *running costs*, which are in some manner dependent upon the quantities of energy delivered over an accounting period; and (2) *fixed charges*, which are in the main dependent upon the capital cost and the useful life of the mains and other plant employed.

Certain categories of costs, such as routine attention, expenses of management, accounting, etc., are usually taken as fixed charges; they are not susceptible to engineering design, or capable of expression in any fixed ratio to either the quantities of energy delivered or to the capital expended on mains and plant. It is not easy to draw the line between running costs and fixed charges in some of these categories. For example, routine attention and inspection, repairs and maintenance are frequently carried out by the same persons. Repairs and maintenance belong to running, routine inspection to fixed charges. With adequate costing records it is possible to make an approximate division of the doubtful items for any past period. Estimates for new areas can be made by the application of experience.

The aim of economical design is to make the total of running costs and fixed charges a minimum for the service rendered. The design therefore has to take into account the relations between the two; in short, how running costs vary with the capital spent on each part of the system. Some of these relations are fairly simple; one can say that it is worth while to spend more on—for example—mains of a larger size than of a smaller size, because the saving of losses will compensate for the higher capital charges; and if the data are adequate, that there is a definitely most economical relation between load and size. In other cases the relations are more complex. In all cases estimates based on forecasts are subject to some degree of uncertainty. Judgment of probabilities has to be used.

The capital charges of the "Fixed Charges" division consist essentially of two components: (a) interest on capital expended, and (b) redemption or depreciation allowance calculated to

repay the capital during the useful economic life of the assets on which it was spent, or within the redemption period of borrowed capital.

The actual rate of interest at which money can be borrowed for a supply undertaking depends upon a number of factors which it is hardly necessary to discuss here; the engineers and financial people concerned can always make a close guess of the rate at which money can be obtained for the contemplated enterprise at the relevant time.

Redemption or depreciation allocations in theory make good the "wearing out" of the capital values of the assets to which they refer. The capital value exhausted by use is offset by the accumulation of a fund which will suffice to replace the assets at the end of their useful life. Assuming the data and calculation to be correct, the periodical allocations for the purpose of this fund are evidently true "costs" of the service rendered by the assets.

In the case of loans raised by local authorities for electrical undertakings, the Electricity Commissioners prescribe a set of periods within which loans in respect of each class of asset must be repaid. The necessary allocations towards such redemption then fix those items of costs. Companies do not, as a rule, show depreciation allocations to specific classes of assets in their accounts. They must make the provision in some way, for the sake of prudent finance, i.e. the maintenance of capital intact. The calculation of a correct depreciation rate requires, first, an estimate of the economic life of each asset, secondly, the calculation of a scale of annual or other allocations which will replace the asset at the end of its useful life. For the present purpose one may take depreciation and redemption allocations as equivalent; but they may not be of equal amounts. The redemption period of a loan may be shorter than the useful life of the assets on which it was spent. It should not be longer. It is hardly necessary here to describe the several ways in which loan redemption is effected. For the present purpose the depreciation allocation may be taken as an annuity which, with accumulated compound interest, will reach the capital value of the asset at the end of its useful economic life, possibly less than its physical life. Tables are available which give the rate of annuity required for any life period and rate of interest which it is estimated can be obtained on the accumulations.

The designer of a distributing system has to make the best estimate he can of the useful lives of cables, transformers, and so on, and of the interest rate likely to be obtainable throughout such periods. Taken conservatively, the data give a safe figure for the annual allocation in respect of exhausted value. The two items, interest plus depreciation, together give an annual allocation of a certain percentage of the capital spent on the asset. The total is an estimate of cost which has to be earned by the use of the asset in order to maintain the undertaking in a permanently sound financial condition.

In the future it may well turn out that the depreciation fund is not spent in replacing the assets by exactly similar ones. It is pretty certain that depreciation funds accumulated during the last forty years, say in respect to certain cables, will not be spent on replacing them by similar cables. That is of no consequence so long as the available amount suffices to buy at least equally useful cables. No one can predict what sort of cables will be required thirty or forty years hence, nor the level of prices at that time.

Obsolescence should not be allowed for in these estimates. If the course of events makes it worth while to replace any asset before its useful life is ended with something better, costing more than the available depreciation fund pertaining to the original article, the difference is a proper capital charge. "Worth while" means that the new asset will have sufficiently superior net earning power at least to pay the capital charges on the difference.

The capital charges bear some relation to the designed load capacity of the asset: not a direct or linear relation, but one which can be expressed as an annual charge per kilowatt of the designed load, and of the actual maximum load carried during each year. Whilst the actual load is below the designed load, the annual cost per kilowatt (or kilovolt-ampere) will be greater than that for the designed load. Maintenance charges are rather proportional to physical magnitude—length of mains for example—than to the load carried; it may be taken roughly as proportional to the product (length of main) into (designed load capacity); but the relation varies with different classes of plant. In some there is a relation between maintenance costs and the magnitude and duration of the load carried.

The total of generation (or bulk supply), transmission, distribution, and management costs, divided by the number of

kilowatt-hours delivered to consumers, yields a quotient of cost per kilowatt-hour, a "statistical average" cost which must be met from consumers' payments if the undertaking is to remain stable and solvent. Actually, the consumers should pay more than the total costs. Whether the margin is devoted to the payment of dividends, the relief of rates, a reserve fund, or is "employed in the business," it can be conveniently called a "profit" margin.

A profit margin is not a cost. It is, properly speaking, an addition to the costs to make up a selling price which will be remunerative.

It will be noticed that a distinction is here drawn between interest on capital, taken as a cost in the foregoing analysis, and a profit margin; which latter can also be expressed as a rate of interest on the capital employed. This reflects the fact that capital cannot be had for nothing, but that the rate of return required varies with different classes of investors. Certain classes require a fixed rate of return with a definite period of repayment, both guaranteed. The prevailing rate at which money can be raised for "gilt-edged" or perfectly safe investment can be properly taken as the minimum remuneration required. That rate varies with the circumstances of the time. The rate at which municipal authorities in good financial repute can raise money with the ultimate recourse of the rates is ascertainable at any time; it is a proper rate to take as an item of cost.

Profit- (dividend-) earning investments appeal to a different class of investors; those able and willing to take the risk of fluctuating returns, and who require the inducement of the probability of a larger average return—at least in the long run —than that obtainable from the gilt-edged type of investment. The distinction is exhibited in the capital structures of companies. A company in good standing can raise capital on debenture secured against its earnings and assets, and redeemable at a stated date, at an interest rate which is considerably lower than the dividend rate expected by its shareholders. The debenture interest and redemption allocations are properly regarded as costs, which have to be met out of revenue before any dividend can be paid. Usually a prosperous company can raise debenture capital on terms approximating to those obtainable by municipal authorities with the backing of the rates. It may be taken as certain that it cannot place

debentures at lower rates of interest than those offered by gilt-edged securities. Even if the whole of a company's capital is raised by the issue of shares, the minimum dividend rate which will be regarded as satisfactory will be something higher than the current "gilt-edged" rate.

That is the reasoned justification for taking an appropriate rate of interest as a cost, quite irrespective of the capital structure of the concern.

The profit margin to be added is not strictly a matter for the engineer designing a distributing system ; he must take into account some minimum rate of interest as an element of cost, because he has to balance capital spent against savings in working expenses in order to arrive at a true minimum of total cost. But he cannot ignore the matter of price altogether. A complete prospective estimate must be based on some assessment of the probable demand and consumption in the area within some given time. That assessment presupposes that the prices charged will be such as to attract the demand so that the margin of profit required for successful financing cannot be omitted from a complete scheme.

So the complete form of an estimate will be, in effect, that, if and when the demand and consumption reach the magnitude assumed, the capital required will be so much, the total annual costs so much, permitting of prices yielding some stated margin ; and that the prices can be expected to attract the calculated demand within a certain time based upon experience, and from a knowledge of such competing agencies as may exist.

There will always be some period during which new works will be unremunerative, though interest on borrowed capital will have to be found during such periods.

Very roughly it may be said that "Running Costs" are proportional to the number of kilowatt-hours delivered, and that "Fixed Charges" are proportional to the maximum kilowatts (or kilovolt-amperes) demand over a given accounting period ; the total costs can be expressed as a periodic cost per kilowatt (or kilovolt-ampere) of maximum demand, plus a cost per kilowatt-hour sold. That division is the logical basis of "Two-part" and "Maximum Demand" tariffs.

CHAPTER III

PHYSICAL EFFICIENCY AND ECONOMIC EFFICIENCY

THE physical efficiency of a distributing system is the numerical ratio between the amounts of energy supplied to the system and delivered by the system to the consumers.

An efficiency of 90 per cent means that for 100 units delivered *to* a line, 90 units are delivered *by* it. The difference is the loss or waste, a part of the cost of the transmission of the 90 units.

One might similarly define the efficiency of the transport of coal by railway or steamer as the ratio of the coal delivered at the end of the journey to that loaded at the beginning, if the coal used on the journey were reckoned as part of the original load.

Costs is a word of narrower general application. It will be used here to mean the expenditure in money value incurred in the distribution of electrical energy, money value being the common denominator to which all the items of expense are eventually reduced in commercial practice.

In reckoning costs it is convenient to take the quantity of the finished or saleable product delivered as the unit. Hence percentages will not have the same value as in the case of efficiency, where the raw product is taken as the unit of quantity. For example, in the coal transport above instanced, the cost of 90 tons of coal delivered is increased by one-ninth, or 11·1 per cent has to be added to the price per ton of the coal loaded to give the cost per ton of the coal delivered so as to provide for the item which may be called "loss in transmission." Other costs of transmission, or carriage, have also to be divided among the 90 tons actually delivered, so that of the total cost of the coal delivered it can be said that a certain percentage is cost of transport. The distinction between the two bases of percentage is important. For example, if in a given system the overall efficiency of distribution is 75 per cent, three units are delivered for four units generated. But the generating costs of the three units delivered are those of four units; or 33 per cent has to be added to the generating costs of the units supplied to the distribution system, not 25 per cent.

The total cost of distribution includes the capital charges and the cost of maintenance of the system. The value of the energy expended in the process of delivery may be taken in

the gross as the cost of producing that energy. In the usual allocation of generating costs *per unit sold*, that cost (the energy expended in delivery) is included in the generating costs; really it is a cost of distribution, and must be taken into account in designing for minimum cost of distribution. On analysis it will appear that the bare cost of generating the energy expended in distribution is not the proper figure to use in the detail design. Taking the case of a distributing system including feeders from the generating station to a number of sub-stations : the cost of the energy delivered by the feeders includes the capital charges on the feeders, i.e. the price of the energy expended in the substation apparatus must be taken at more than the price of that expended in the feeders. The cost of the energy expended in the distributors must be taken at another still higher rate which includes both the losses and the capital charges involved by transformation and/or conversion in the substations. At each stage the energy sent forward has an increased unit cost. Any loss caused by under-registration of the consumers' meters should be priced at the selling rate to the consumers.

The economic problem is to find the distribution design or lay-out which will deliver the energy sold at a minimum cost when all the elements of cost are brought into account.

In the simplest case of transmitting a given quantity of energy from point A to point B at certain time rates, without intermediate transformation, the physical efficiency involves only the losses in the conductors; the economic efficiency involves the capital costs on the mains as well.

For such a point-to-point transmission, if the time distribution of the delivery is known, the problem "What is the most economical size of the conductor?" is capable of exact solution for a given set of the other relevant data. In the case of general distribution to many points (consumers) certain data can be obtained for an existing system by sufficiently detailed measurements; for a prospective system they can only be estimated with such assistance as may be available from experience in similar areas, modified by any special features of the area likely to affect the magnitude, time incidence, and location of the individual demands.

Conductor Efficiency and Costs. For continuous current, assuming no leakage, the current at the delivery end of a pair ("go and return" wires) is the same as at the source; the

current efficiency may be called 100 per cent. The losses are shown by a drop of voltage at the delivery end; numerically that drop is equal to the product of the current in amperes and the resistance of the conductor pair in ohms. This is usually symbolized as $IR = V$, I standing for amperes, R for ohms, and V for volts drop. Since the energy put in and delivered is proportional to the product of the volts and amperes at the respective ends, the energy efficiency is the ratio of the voltage delivered to that put in. That is, the energy transmission efficiency of a conductor pair depends upon the voltage. For example, if the pair has a resistance of $\frac{1}{2}$ ohm and carries a current of 100 amperes, the drop of voltage will be 50, the energy loss $100 \times 50 = 5\,000$ watts. If the voltage at the source is 100, that at the delivery end will be 50; the energy delivered will be 5 000 watts, equal to that lost in the conductors, and the efficiency of transmission 50 per cent. If the initial voltage is 1 000, the delivery voltage will be 950, and the efficiency of transmission 95 per cent. The amount of the loss remains the same—5 000 watts—but as the energy put in is 100,000 watts, it is only one-twentieth of that, instead of one-half as in the 100 volt case. Transmission efficiency is therefore not an inherent property of the conductor, which has an inherent loss factor, its resistance. The voltage drop is IR; the energy expended or "lost" is I^2R. For a given initial voltage, the efficiency of a conductor is inversely proportional to the current flowing through it: the efficiency is zero when the product IR equals the initial voltage, and approaches 100 per cent when the current approaches zero; the utility of the conductor then also becomes zero. The load versus efficiency curve shows zero efficiency for the short-circuit condition which absorbs all the initial voltage; the commercial efficiency is zero for both no load and short-circuit conditions.

As the resistance of a unit length of a conductor of a given material is inversely proportional to its cross-section, and its cost if bought by weight is directly proportional to that section; the electrical losses and the capital charges are affected in opposite directions by a change in ratio of current and of cross-section. A smaller conductor will be cheaper and cost less in capital charges, but it will have a higher rate of energy loss for any given current, and vice versa.

Then there must be some relation between the cost of the conductor and the cost of energy—in other words between the

size of the conductor and the current it carries—which makes the total cost of transmitting the current a minimum. The ratio of current to cross-section is denominated *current density* and expressed as *amperes per square inch* or *per millimetre square*. American engineers use the *circular mil* as the unit of cross-section. They call a conductor of 1 in. in diameter, one of one million circular mils; whereas in British parlance it would be called of 0·7854 in.²

Lord Kelvin showed long ago that the condition for minimum cost is that the capital charges over any period and the value of the energy losses over the same period are equal. This is known as the *Kelvin Law*.*

Accepting that law, and converting the money cost, capital charges per annum, say, into the kilowatt-hours which cost that amount, the annual cost of a conductor is made up of two terms both involving the cross-section, or the weight per unit length. The capital charge term can be written as ma; and the losses term as nI^2/a, where a is the cross-section, and I the current; the condition of minimum cost then is—

$$ma = nI^2/a$$

and the solution for current density is—

$$\sqrt{(m/n)} = I/a.$$

m is a cost per annum expressed as kilowatt-hours; it is the product of the number of kilowatt-hours costing one unit of money and the annual money cost of a unit cross-section of conductor of unit length, it can therefore be written—

$m = (wpR \times$ weight of unit length of unit cross-section);

n is the annual loss in kWh. in the same conductor that is—

$$n = I^2r/1\,000 \times \text{hours of use.}$$

where

$w =$ number of kWh. costing one unit of money;

$p =$ cost per unit of weight of the conductor;

$R =$ annual rate of interest, etc., on the cost;

$r =$ resistance in ohms of unit length of conductor of unit cross-section.

Taking copper as the conductor, unit cross-section as 1 in.², unit length as 1 mile, £1 as the unit of money; the weight in

tons T as 9·1 the resistance in ohms as 0·045, and the period as 1 year (8 760 hours) ; the ratio m/n becomes—

$$m/n = wpR \times (T/8\text{·}760r)$$

(the divisor 8·760 reduces I^2r, which is in watts, to the rate of loss in kW.).

$$T/8\text{·}76r = 9\text{·}1/(8\text{·}76 \times 0\text{·}045) = 23\text{·}06,$$

hence (the square root of 23·06 being 4·802) the current per square inch is

$$\sqrt{(wpR)} \times 4\text{·}802.$$

The constant 4·802 is a numerical value depending only upon the physical properties of the material, and the period, one year, taken for the capital charges.

Hence the most economical current density is proportional to the square root of each of the factors : w, the kilowatt-hours costing one unit of money ; p the cost in money of unit weight of the conductor ; and R the rate or fraction of that cost representing the capital charges. That is a general expression for any units. The numeric 4·802 is the multiplier for copper and for one year as the period over which the capital charges accrue and the current passes.

For money values in pounds, $w = 240/d$ where d is the cost in pence of 1 kWh. If, for example, the cost of 1 kWh. is 1d. $w = 240$; if the cost per kWh. is 0·5d., $w = 480$; so that the cost per ton of copper and the rate of capital charges being fixed, the most economical current density is proportional to the square root of the number of kilowatt-hours costing £1, or any other money unit employed ; or it is inversely proportional to the square root of the cost per kilowatt-hour.

As a simple example take—

Cost per kWh. = 0·5d.

Cost per ton of copper = £100.

Rate of capital charges per annum 10 per cent or 0·1 ; then— $wpR = 480 \times 10 = 4\ 800$, and the current per in.^{2}.

$(\sqrt{4\ 800}) \times 4\text{·}802 = 69\text{·}28 \times 4\text{·}802 = 332\text{·}8$ amperes.

If the kWh. costs 1d., wpR is 2 400 and the current is

$(\sqrt{2\ 400}) \times 4\text{·}802 = 48\text{·}98 \times 4\text{·}802 = 235$ amperes ;

the last value being equal to the first divided by the square root of the ratio 1/0·5, viz. 1·414.

The voltage drop per mile is evidently Ir, or $I \times 0\text{·}045$ for copper, and is a constant for that current density.

The kilowatt-hours of losses and the capital charges per mile

of 1 in.^{2} of copper can be reduced to losses and cost per ton by dividing by 9·1; this gives an annual rate of loss per ton at the calculated current density, independent of length or cross-section. Numerically this is $I^2 \times 0·0433$, where I is in amperes per square inch.

Any given "cost per ton per annum" corresponds to the product of many pairs of "price per kilowatt-hours" and "number of kilowatt-hours per annum." The curve of constant cost against co-ordinates of those quantities is therefore a rectangular hyperbola, the equation to which is $XY = $ constant $= £ \times 240$, X being scaled in pence per kilowatt-hour, and Y in kilowatt-hours. Such a curve is shown in Fig. 1.

On the same figure is drawn a curve of losses per ton per annum for current densities between 100 and 1 000 A. per in.^{2}. A horizontal line drawn from the intersection of any pair of values of "annual cost per ton" and "pence per kilowatt-hour" cuts the current density curve at the most economical value for 100 per cent load lactor.

Fig. 2 is a nomogram expressing the same relations, scaled so that a line drawn through any pair of values, pence per kilowatt-hour on scale *A* and £ per annum per ton on scale *B*, cuts scale *C* in the corresponding value of kilowatt-hour, per annum and the economic current density, also figured on scale *C*.

The following table may be found useful—

TABLE I

Current Density (A. per in.^{2})	Loss (kWh. per annum per ton of copper)	Value of Loss per annum (£)	Volts drop per 1 000 yd.
100	433	1·805	2·56
150	975	4·062	3·84
200	1 732	7·215	5·11
250	2 700	11·250	6·37
300	3 900	16·250	7·67
350	5 300	22·080	8·95
400	6 930	28·87	10·23
450	8 771	36·55	11·51
500	10 830	45·12	12·29
600	15 600	64·99	15·36
700	21 200	88·32	17·90
800	27 730	115·55	20·46
900	35 080	146·20	23·11
1 000	43 300	180 50	25·57

Note. Figures are only to slide rule accuracy. The second and third columns are for 100 per cent load factor.
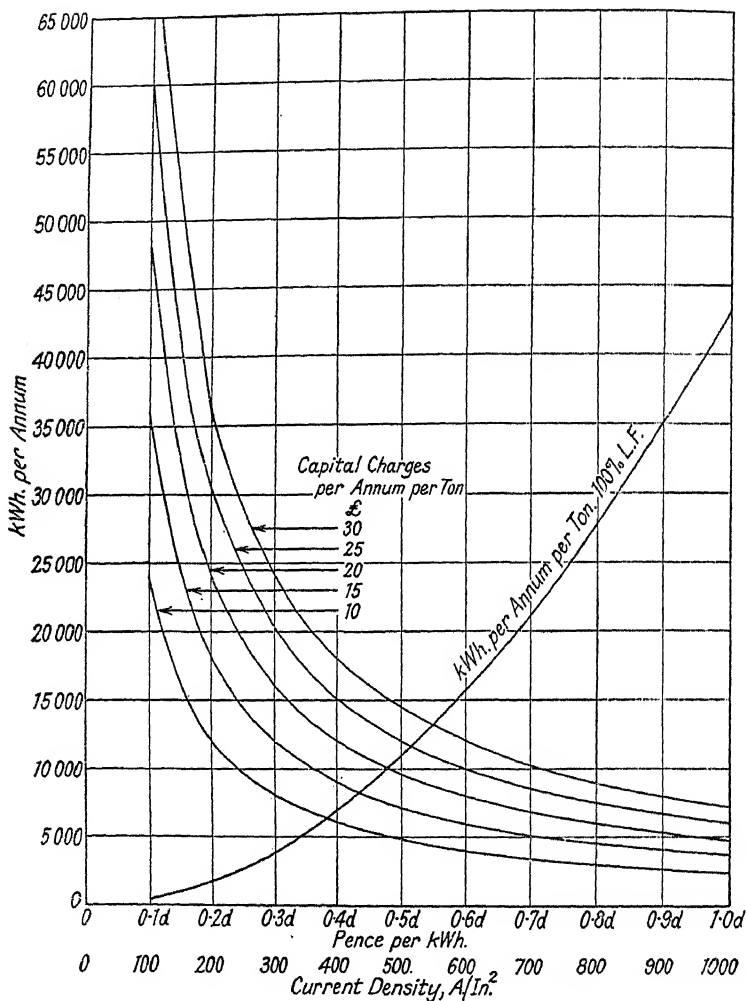
FIG. 1. CURVES RELATING CAPITAL CHARGES PER ANNUM AND
KWH. PER ANNUM PER TON AT 100 PER CENT LOAD FACTOR

Annual losses in kWh. per ton = $I^2 \times 0.0433$ where $I = A$. per in². at 100% l.f.

Equivalent kWh. per annum = £ capital charges $\times \dfrac{240}{\text{price per kWh.}}$ per ton.

A horizontal line cutting any of the £ per annum curves at any ordinate of pence per
kWh. cuts the current density curve at the most economical value of current density indicated
by the ordinate at that point for 100% l.f.
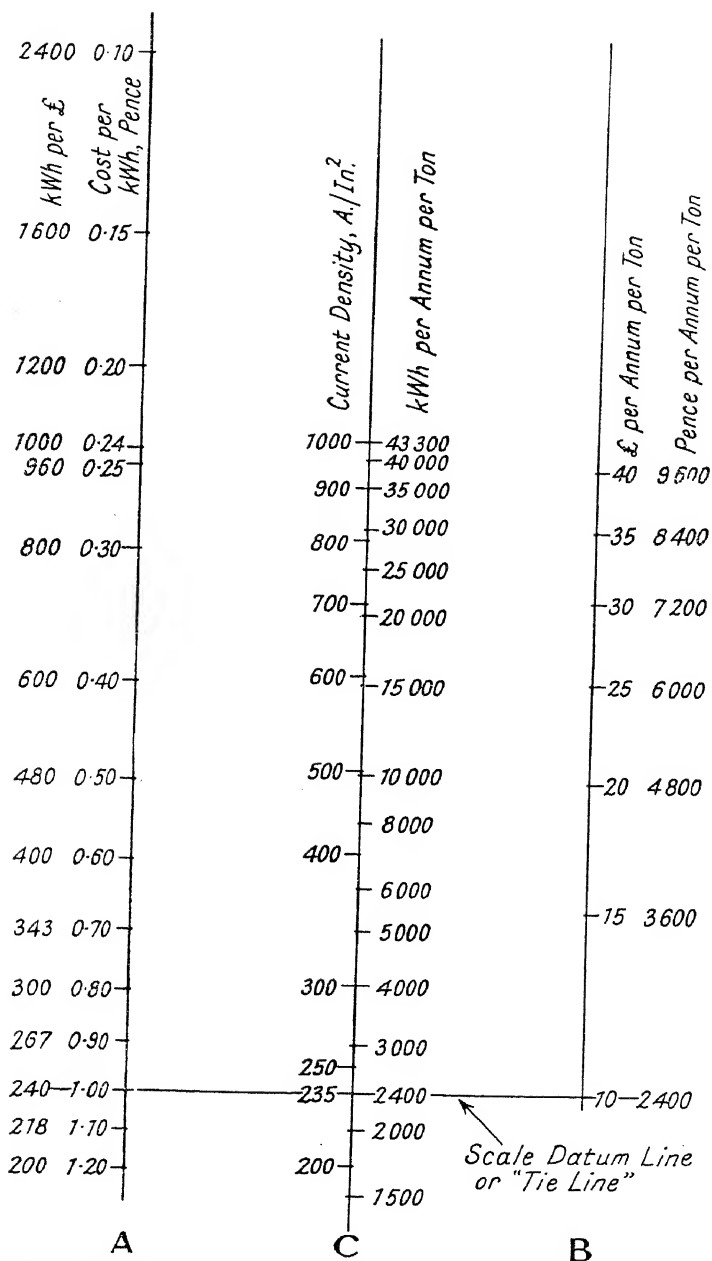
FIG. 2. NOMOGRAM EXPRESSING THE RELATIONS SHOWN IN FIG. 1

A useful mnemonic is that at 400 A. per in.^{2} the voltage drop is 10 volts per 1 000 yd., nearly enough for most purposes.

The "Kelvin" condition is not an expression for physical efficiency but for the most economical utilization of the conductor material of the resistivity and cost considered. It is a current density independent of the length or of the size of the conductor, so long as the cost of the conductor is proportional to its cross-section, or weight per unit length. In a more generalized form, a conductor is of the most economical size for a given load when the losses in transmitting that load cost as much as the capital charges on the conductor during the complete loading cycle—usually one year.

It is necessary to make very clear the relation between the load distribution in time—i.e. the load diagram over a year or other load cycle period—and the most economical current density.

If the load is not uniform throughout the year (if that period is taken, as is usual), the relation of equal money value of the losses and the capital charges still holds good as the condition of minimum costs; but the current density which satisfies that condition will be different, whether it is reckoned on the average current over the year, or the maximum current at any time. This can be expressed as follows: for a given load diagram the copper section which results in losses equal in value to the capital charges dependent on that section is the most economical, whatever the shape of the load diagram. Since the losses at any load are proportional to the square of the current, over a given period the losses will be proportional to the mean square of the current during that period. The current density found for the uniform current condition is therefore the square root of the mean square of a variable current which will give the same total loss. The ratio of the square root of the mean square to the average value of the current is known as the *r.m.s.* value or *form factor*. It is the multiplier for the copper section for the average current; its reciprocal is the multiplier for the current density also reckoned on the average current, the standard of reference being the current density (or copper section) found for a uniform whole time current.

Symbolically, if I/a is the economical current density for a uniform whole time load, the average current density for a variable load will be: $I/a \times 1/I_{r.m.s.}$; and the maximum

will be $I/a \times 1/I_{r.m.s.} \times 1/$load factor, where load factor is reckoned in the usual way as the ratio of average to maximum load.

The load factor is not directly related to the economical current density. A load factor figure of 50 per cent or of 25 per cent gives no more information about the form factor of the load than that it may have a certain maximum value, viz. the square root of the reciprocal of the load factor, e.g. $\sqrt{2}$ or 1·414 for a load factor of 50 per cent, or $\sqrt{4}$, or 2, for a load factor of 25 per cent, and so on. Such maxima values are only reached when the load factors are time factors, i.e. denote the fraction of the whole time over which there has been a uniform load, with no load for the remainder of the time.

A load factor of 50 per cent is given equally by a load diagram which is uniform for half the time and nil for the other half; and a triangular load diagram reaching a maximum and coming back to zero having as time-base the whole period. But the r.m.s. value of the rectangle on half the time is 1·414, and of the triangle on the whole time is 1·155. The relative current densities for the average current over the whole time will therefore be 0·707 in the first, and 0·866 in the second case.

Fig. 3 and Table II illustrate the facts. Fig. 3 shows eight load diagrams all having a load factor of 25 per cent, i.e. the maximum load or current is four times the average. Table II gives descriptions of the time distributions as over one day, the mean square factor, the r.m.s. or form factor multiplier of the copper section proper to a constant load equal to the average, the reciprocal current density multiplier, the voltage drop at the maximum load, the generating capacity needed to provide the peak load losses, and the daily loss in kilowatt-hours. This table is drawn up for a daily delivery of 12 000 kWh., or an average load of 500 kW., at a constant voltage, on the assumption that the most economical section for a uniform load produces a voltage drop of 5 per cent of that delivered, or a daily copper loss of 600 kWh. A 25 per cent load factor means that the peak load is four times the average—or 2 000 kW.—so that at maximum load the voltage drop is four times, and the copper loss sixteen times, what they would be for the average load, if the same conductor were used in both cases. The r.m.s. or form factor value of the variable loads is the multiplier for the unit or standard section for the average load, therefore the multiplier for the capital charges on that section. The last

TABLE II

CONDUCTOR SECTIONS AND CURRENT DENSITIES FOR MAXIMUM ECONOMY

according to the Kelvin Law, for Load Diagrams of several forms all giving a Load-factor of 25 per cent, i.e. the top load four times the average load, as shown in Fig. 3

1	2	3	4	5	6	7	8	9
No. of Load Diagram	Description of Load Diagram	Mean Square	R.M.S. or Form Factor (multiplied for conductor section)	Reciprocal of R.M.S. (multiplied for current density)	Voltage drop at Top Load (per cent)	Load Factor of Losses (per cent)	Losses per day kWh.	Loss Rate at Top Load kW.
1	{12 hours triangle {12 hours no load	2·66	1·63	0·6125	12·25	16·6	978	245
2	{14·4 hours 0·25 average { 9·6 hours triangle	2·313	1·521	0·656	13·20	14·5	913	263
3	{16 hours 0·4 average { 8 hours triangle	2·084	1·44	0·693	13·90	12·95	864	278
4	{17·143 hours 0·5 average { 6·857 hours triangle	1·917	1·384	0·723	14·46	11·97	830	278
5	{18·353 hours 0·6 average { 5·647 hours triangle	1·67	1·292	0·713	15·56	10·41	775	310
6	{21 hours 0·8 average { 3 hours triangle	1·386	1·177	0·848	16·96	8·65	706	340
7	{22·452 hours 0·9 average { 1·548 hours triangle	1·197	1·094	0·915	18·50	7·47	656	366
8	{ 18 hours no load { 6 hours rectangle { at 4 times average	4·00	2·00	0·500	10·00	25·00	1200	200

The values in this table are based upon a most economical current density and conductor section for 100 per cent load factor giving a drop of 5 per cent of the delivered voltage, both taken as unity.

The daily losses and loss rates at top-load (Columns 8 and 9) are based upon a daily delivery of 12 000 kWh. in every case. The loss rate is the generating capacity needed to supply the losses at the top load, keeping the delivery voltage constant: the top load being 4 000 kW. in each case. For the equivalent constant load of 500 kW., the 5 per cent loss assumed would result in a daily loss of 600 kWh.

The cost per kWh. of the losses has been taken as constant. That may not be true; the capital charges on generating capacity per kilowatt-hours rise as the load factor of the losses falls. The simple Kelvin law should be modified to allow for that: the effect is that the conductor sections should be larger and the current densities smaller as the load factor of the losses falls, to give the most economical values.

Explanatory Notes

Column 2. "Hours triangle" means the time base of the peak load, the load for the remainder of the 24 hours being uniform. Diagram No. 8 has a steady "rectangular" load for 6 hours and no load for 18 hours.

Column 3. Mean squares of the load diagram ordinates.

Column 4. Root mean Square or Form-factor, which is the multiplier of the unit conductor section for each diagram.

Column 5. Reciprocal of the form-factor, which is the multiplier of the unit current density referred to the average current, for each diagram.

Column 6. Voltage drop at top load as a percentage of the delivered voltage.

Column 7. Load factor of the losses, referred to the losses at top load.

Column 8. Losses per day for 12 000 kWh. delivered.

Column 9. Loss rate, or generating capacity needed to supply the losses at top load; based upon 25 kW. at 100 per cent load factor. The two last columns, 8 and 9, alone refer to definite magnitudes of load and output; the values in the other columns are ratios of general application to load diagrams of the forms considered, irrespective of their scales.
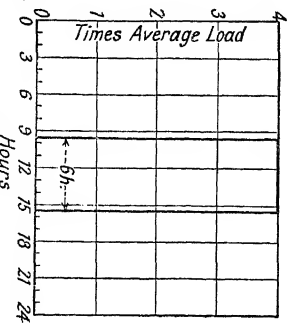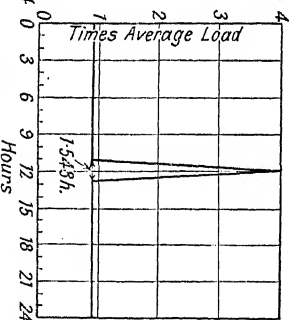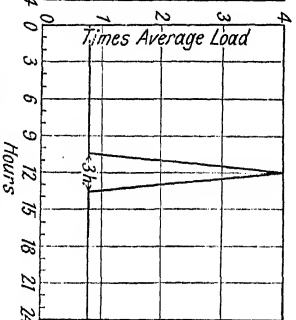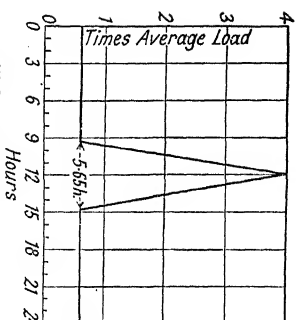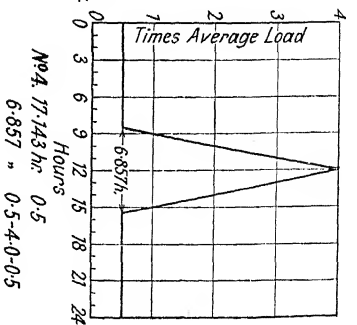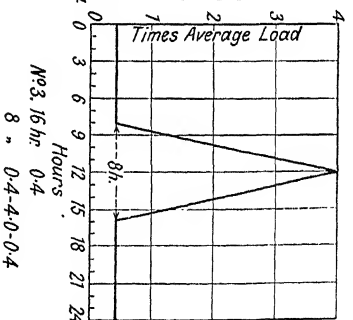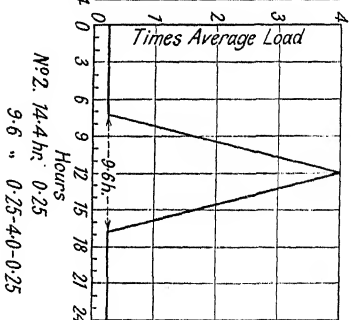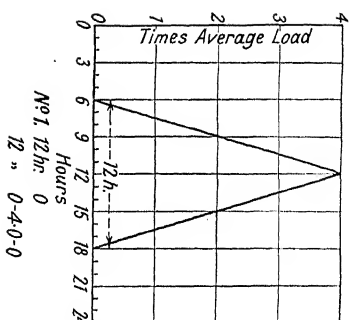
Fig. 3. Eight Load Diagrams for One Day

All of 25% load factor: identical maximum and average loads.

column gives the values of the daily losses which equal the product of the form factor and the 600 kWh. loss for a uniform load, showing that the condition of the equality of the two quantities is fulfilled.

If this reasoning is not self-evident, it would be a useful exercise to try the result of using any other copper section than the "r.m.s. $\times$ uniform load" section for any of the load diagrams of Fig. 3.

In a practical case one must take or estimate a load diagram for the whole year; it may be a chosen typical load diagram, or a composite "weighted" aggregate of selected daily diagrams.

The table shows some other points of interest in addition to the root fact that the cost of transmission of a given quantity of energy is a minimum when the rate of transmission is constant, and is increased by any deviation from that constancy. The generating capacity must be sufficient for the maximum load plus the maximum rate of loss at that load; the generating plant needed for that loss is worked at a lower load factor than that of the delivered load (excepting in the limiting cases where the load is uniform for the fraction of the whole time expressed by the load factor; when it has the same value). That means that the capital charges upon that part of the generating plant providing the maximum rate of loss are greater per kilowatt-hour than upon that part providing the delivered energy. Hence in any particular conditions those capital charges should be considered, when it may appear that it is worth while to provide more copper than the section calculated upon a uniform cost per kilowatt-hour generated. This can be allowed for by pricing the losses at an appropriately higher rate. If, for example, the cost of the energy delivered to the main is £m per kW. per annum, plus n pence per kWh., the price of the losses is—

$$240m/8\ 760 + n;\ \text{or}\ (0{\cdot}0274\ m + n)\ \text{pence}.$$

(If energy is being bought in bulk, as from the C.E.B. Grid, the peak load costs may have to include the kilowatt element of the tariff in full or in part, instead of the generating plant capital charges represented by £m.)

Returning to the general expression for minimum cost of transmission, it will be seen that if the cost per ton of conductor is increased, the rate of loss to equal the charges on it is also increased, that is, a more expensive type or size of cable should

be worked at a higher current density than a less expensive cable. Conversely, if the cost of energy is increased, a cable of given cost per ton should be worked at a lower current density; the reverse applying in both cases.

The "cost per ton" involved means all the costs which depend directly upon the section or weight per unit length of conductor. In the case of cables for laying underground the cost of laying will not usually be much affected by the size of the cable; it may be taken as a constant per unit length. For a given type of cable for working at a given voltage, the cost per ton of copper will vary with the size: smaller cables cost more per ton. It may be found that over some small range of sizes the cost per ton varies little, and a current density applicable to all that range can be found. More generally there is a number of standard sizes of cable, the copper cost of which varies appreciably between the sizes. In such cases the economic current for each size can be found by applying the rule that capital charges and cost of losses should be equal for each size; instead of finding the cable size for a given load, one finds the economic load for each cable size.

Strictly, in selecting cable size from among a number, the "cost per ton of copper" is the difference between the prices of the cables divided by the difference between their copper weights, for equal or unit lengths.

In deciding upon the most economical voltage to employ in a given transmission, one has a number of possible standard voltages to select from. The cost per ton of copper rises steeply with increased voltage. It is simple to work out the economical loading of cables for the several voltages. Obviously the current for a given load is inversely proportional to the delivered voltage. Because high voltage cables are more expensive per ton of copper, the current density should be higher than in low-tension cables. High voltage cables, however, run at a higher temperature for equal current loadings than low voltage cables; also the high voltage cables are subject to heating from dielectric losses which is negligible in low voltage cables.

These two facts combine to limit the application of the Kelvin law; the practical loading of high voltage cables will in general be lighter than that of lower voltage cables; the copper must be less effectively utilized in order to preserve the life of the cable. Which implies that there is some economic limit to the voltage for any given relation between the cost and the working

voltage of cables. Another limitation to the application of the law is the permissible drop of voltage. This is particularly effective in distribution systems. In this country the Regulations prescribe a maximum variation of 6 per cent above and below the "declared pressure" at the consumers' terminals. In practice this means that at maximum load the consumers nearest to the source of supply may have 6 per cent over, and those most remote may have 6 per cent under, the declared pressure. This is a distance limitation of current density.

The Kelvin relation is valid for a given set of the relevant conditions, in respect to the current loading of mains. Its application in practice is limited by permissible heating and permissible voltage variation; either of which may become operative irrespective of the other. Despite the limitations, it is a useful "yardstick," as it will indicate the cost of having regard to the limitations.

It has been said above that the cost of laying a cable does not come into the Kelvin calculations. But it may come in forcibly in the design of a system. A striking example was given in a recently published discussion of the most economical way of increasing the capacity of the transmission system supplying substations in Chicago. The problem resolved itself into the choice of the most economical voltage of transmission. The choice fell upon 66 kV., the deciding factor being that there was in existence a system of spare ducts of 4 in. bore. The 66 kV. cables could be drawn into those ducts. Any lower voltage cables of the required capacity would have been too large, so new ducts would have had to be laid at great expense.

Overhead lines have not been mentioned so far. Assuming that the cost of supports, insulators, etc., is independent of the size (i.e. the weight) of the conductors, the Kelvin relation holds. As a general rule, the number of supports, length of spans, etc., is decided by what may be generalized as the topography of the route, which includes a good many factors of the nature of safety regulations, regard for amenities, etc. An overhead line has to be designed as a mechanical structure. It is useful to work out the Kelvin section for the standard voltages available. The conductor size determines the stresses which come upon the supports. Usually there is some minimum size of conductor imposed by mechanical considerations; that will give a minimum value of the voltage at which the line must be worked in order to afford the required transmission

capacity. The voltage also affects some items of the cost of insulators and supports.

It has to be observed that the discussion in this chapter has dealt only with the ohmic losses in conductors. It is strictly applicable to conductors for continuous current. With alternating currents the losses are greater for equal effective current. With alternating currents the losses are greater for equal effective currents, because the conductor resistance is greater; first, on account of "skin effect" which makes the current density greater near the periphery than in the centre; secondly, on account of "proximity effects," parasitic e.m.f.'s induced by currents flowing in nearby conductors; for example, in a three-core cable each conductor has some inductive effect upon its two neighbours. These effects are approximately calculable, and should be reckoned as additions to the d.c. resistance.

The effect of the load current being out of phase with the voltage only means that the load current has to be reckoned from the kilovolt-ampere load, not from the kilowatt load. The multiplier is given at once by the reciprocal of the power factor of the load; e.g. the current for a load of 0·8 power factor is 1·25 times that for the same kilowatt load at unity power factor.

The heating due to dielectric losses in high voltage cables is continuous so long as the cable is under pressure. It is important in cables working at over about 25 kV. as a factor in the temperature rise which limits the loading, *as if* an addition to the load current were always present.

Extra high voltage cables, for 66 kV. and over, are usually made as single cores, the three of a three-phase system being laid in close proximity. The losses, heating, and most economical loading involve problems of greater complexity than are susceptible to Kelvin law computation; they are beyond the ambit of this chapter. As each case of the use of such cables is of sufficient magnitude to justify detailed study from the economic point of view, no more can be said here.

Applications of the principles set out in this chapter will be made in later discussions.

Transformers. The physical efficiency of a transformer with respect to load follows quite a different law, or shape of curve, from that of a conductor. The losses are conveniently described as *no-load* and *load* losses; or with some lack of exactness as

iron and *copper* losses. The iron losses are continuous so long as the transformer is excited by connection to live mains. The copper losses are roughly proportional to the square of the load current. There is a small element of copper loss in the no-load losses; the iron losses are not entirely independent of the load.

At no-load a transformer has negative efficiency; it wastes the iron losses plus the exciting current copper losses, with no return. The maximum physical efficiency occurs when the two losses are equal. The ratio of constant (iron) and load (copper) losses which will give the maximum commercial efficiency over—say—a year depends upon the form of the load diagram. This ratio of output to input giving the minimum cost per kilowatt-hour of the output also involves the capital charges, i.e. the cost and life of the transformer. The design for minimum cost of a transformer for a given maximum load, and for maximum commercial efficiency on a given load diagram, both involve the proportions of the iron and copper, having regard to the relative costs of those materials. The designer therefore has to make a compromise to produce a transformer which will have the minimum sum of capital charges and value of losses when operating under defined loading conditions.

A transformer which is to be used on a load which will be small for a great part of the operating time and at a maximum for a small part, should have small iron losses relatively to the full load copper losses. A transformer to be used at continuous full load should have relatively small copper losses. The maximum physical efficiency of a transformer results at a load at which the two losses are equal. But that condition means that the transformer is unduly costly for its output. In other words, the loading for maximum physical efficiency is much below the loading for maximum commercial efficiency.

There are other considerations which must be regarded; heating and regulation for instance. The voltage drop between no-load and full load is mainly determined by the copper losses. The maximum temperature attained in working is a product of the duration and magnitude of the load losses, having regard to the cooling arrangements. The maximum load which can be safely carried by a transformer is limited by the temperature rise of the copper which can be endured by the insulating material. Hence the transformer for given

loading conditions, regulation range, and safe working temperature, which will have the minimum annual cost including capital charges, can only be designed to a definite specification of the conditions, including the cost per kilowatt-hour of the losses.

Fortunately the modern transformer has only small full load losses of the order of 2 per cent of the input. Regulation depends partly upon the power factor of the load, one of the items to be specified, since "full load" is defined by kilovolt-amperes rather than by kilowatts. Where transformers are in substations which are attended or can be visited daily, no-load losses can be minimized by switching out some of them during low load periods. Whether this is worth while can only be determined from balancing the value of the saving against the cost of the attention, or of the remote control switching.

The choice of transformers for maximum commercial efficiency therefore involves more complex calculations than the choice of conductors. But since transformers can be added to, or changed for larger (or smaller) ones, whenever load changes make that desirable, the initial choice in a given case does not commit the future so definitely as the initial design of mains which can only be varied at heavy cost. So one may justifiably put down transformers to serve the probable load for a few years ahead; and only buy more or larger ones when the need arises.

The British Standards Institution has issued specifications for transformers; and the British manufacturers have standardized designs from which most requirements can be met. It will rarely be economical to specify requirements departing so much from the standards that special designs have to be made.

Regulating Apparatus. Some form of regulating apparatus is nearly always required in order to maintain the voltage delivered to consumers within the prescribed limits. Regulation involves losses in the regulator; i.e. it adds to the losses in the conductors and transformers. Essentially, all regulators (outside of regulators of generator voltage) are transformers; they draw current from the mains—ultimately from the generating source—partly to make up for the drop losses in the distributors beyond them, partly to supply their own losses; hence the losses in the distributors cost more per kilowatt-hour than without regulators, on account both of the added

losses and the capital charges on the regulators. This enhanced cost should be taken into account in calculating the most economical current density in the distributors. If in any given case a regulator can be dispensed with by using a lower current density in the distributors, one can put the additional capital charges of the larger distributors, less the value of the lower losses in them, against the capital charges, losses, and cost of attendance of the regulator in order to see whether or no the regulator is an economic proposition; or can calculate the most economical combination of conductor sizes and regulator range.

It is a truism to say that all the losses in a system operate so as eventually to claim some share of the capacity of the generating plant. Losses in regulators are at a maximum when the losses in the regulated mains, are at a maximum. From the point of view of the generating station—or bulk supply point— such losses are the peak of the peak load, employing generating capacity at the lowest load factor, and therefore very expensive in station capital charges or the equivalent bulk supply demand charge. Hence the losses in regulators are of an expensive character on all counts. It is possible that a livelier appreciation of these truisms would result in a more liberal design of conductors, especially in low-tension distributors, than is sometimes practised.

As to the physical efficiency of regulators, that can be calculated in the usual form as the ratio of output to input. It is clearly desirable to have high efficiency at the maximum "boost" because peak load losses are expensive (also because such losses take the form of heat); but the no-load losses, which are continuous unless the regulator can be switched out for part of the time, are not negligible.

Commercial efficiency is a more complicated matter. The capacity and cost for a given load depend upon the magnitude of the maximum boost, i.e. upon the sizes and lengths of the conductors serving the load.

For large loads, tap-changing transformers are probably the cheapest form of regulator; usually they are only economical for step-up and step-down transformers at generating and grid substations. For detail distribution several forms of auto-transformers, "tail-end" boosters, and induction regulators are available; the choice depends upon the detail conditions in each case. Inquiries for quotations should specify the duty required in the fullest way.

In all calculations for economical regulation, the load and form factors of the load are essential data.

Switchgear. The physical efficiency of switchgear is rarely of importance; other considerations impose conductor dimensions which make the conductor losses negligible. Switchgear in itself does not earn anything; the services it performs are hardly capable of precise money valuation, but failure may be very costly. Hence capital cost, reliability, and maintenance cost are the prime things to consider; space occupation is often important. Since the function of switchgear is to afford facilities for altering the connections of lines and apparatus, the smaller the number of changes required the simpler can be the switchgear. Hence the first point in design is to settle the minimum number of changes essential in any position. A good deal of ingenuity may be profitably employed in making the necessary changes with a minimum number of switches. This applies particularly to the lay-outs at interconnecting points for alternative ways of feeding and splitting up networks. Probably almost all the functions of switches on distributing systems at and below 11 kV. can be satisfactorily-performed by switch-fuses of the enclosed type, with arc-extinguishing filling such as carbon tetrachloride. But if on-load change-over switching is required at any points, a circuit-breaker must be employed—oil-filled or the equivalent, to suit the voltage. The switch-fuse, it may be noted, is an overload protective device, as well as a switch.

In the initial design of a system quite simple switching arrangements may meet immediate needs. But it will be wise to consider the future, adopting such a lay-out that additions can be made without disturbance of the original arrangements, e.g. by providing bus-bars, capable of accommodating additional switch units, or easily extended by bolting on new lengths.

It is a good plan to standardize switch lay-outs, so that those at different substations differ only in the number of switch units and bus-bar sections. There are limits to the extent to which this can be carried out.

Makers offer standardized units from which it should usually be possible to choose models applicable to all ordinary purposes.

Protective Gear. Other than fuses, overload and earth-fault trips on h.t. switches, protective gear is rarely needed on

distributing systems. It may be necessary on ring mains; certainly if these work at over 11 kV. The economic value of protective gear is that of insurance against (*a*) the extension of the interruption of supply beyond a faulty section, and (*b*) the extension of damage beyond the originating fault. Data for estimating the probability of such occurrences and the cost of the consequences are not available in sufficient quantity to allow of any accurate calculation of the risks and the appropriate cost of insuring against them. Nothing more can be said here than a recommendation to refer to such records of experience as may be available; with the addition that on overhead systems there is an imperative necessity to make "dead" any conductor broken by storm or accident, for the protection of life and property.

Capital Charges. The calculation of the most economical current density in cables involves the annual charges as a percentage of the cost of the cables. As already stated in the last chapter, these are made up of two components (*a*) interest on the capital, and (*b*) an annuity to provide for the cost of replacement at the end of the useful life, i.e. depreciation. The determining conditions for (*a*) have already been discussed. The second component (*b*) includes elements of prediction. One of these is the useful or economic life of the cable; strictly, for how long the cable can be expected to carry its estimated load without excessive costs of maintenance, or unreliable operation due to deterioration. Another element is the probable cost of replacement; yet another is the interest rate the accumulated annuity can be expected to earn. If it is assumed that the life of the cable will be so many years, that the cost of replacement will be the same as the original cost, and that a certain rate of interest can be reckoned on; the calculation of the annuity which, accumulated and invested at compound interest, will amount to that sum is simple and straightforward.

None of these elements can be known exactly. The useful or economic life of a cable is of the order of thirty to forty years (that is, for cables of up to perhaps 22 kV. pressure; there is not yet enough experience of extra-high-tension over that to go upon). The cost of replacement will be abated by the value of the scrap recovered—mostly copper—an uncertain amount; the rate of interest obtainable by the accumulated depreciation is also liable to some measure of uncertainty. The safe way to

discount these uncertainties is to estimate the life conservatively and to reckon on a low "gilt-edged" rate of interest. The main point is to be on the safe side. The course of events during the life of the cable will determine what the actual annuity rate should be; it can be corrected at any time when it seems to be too high or too low. The cost of energy is likely to alter in the course of the life of the cable. One can only adopt reasonable values for these elements in the circumstances of the time.

Part of the cost of replacing a cable is that of laying it. That is not a function of the size of the cable and does not come into the calculation of economic density. It has to be provided for by a percentage of the original cost of laying over the life. If the cable is drawn into permanent ducts, there will be little or no charge for trenching and making good; the ducts may be reckoned to have a quasi-permanent value, and the depreciation rate of the laying cost taken at a very low percentage. The original cost of laying will be higher, the interest component of the annual charges higher, and the depreciation component lower, than if the cable is laid direct in the ground. These are not the total of the considerations for and against ducts; but where the cost of breaking up and making good of street paving is high, they may be sufficient to decide the choice.

In practice a cable may have to be replaced long before the end of its useful life. It may become inadequate for the load upon it, or may be displaced by a change of system. It must be assumed that the replacement will be made because it is expected to be remunerative; hence any cost in excess of the depreciation fund available at the time is strictly new capital expenditure.

Much the same order of considerations apply to the estimates of capital charges on transformers and other distribution apparatus. The lives of such things are usually shorter than the lives which can be safely reckoned for cables; but on the other hand the costs of handling, etc., are much lower costs than those pertaining to the laying of cables. Small transformers can be used elsewhere after replacement by larger ones; that is not usually practicable with cables taken out because they have become overloaded.

In practice the engineer designing a distribution system does not separate the annual charges into the components above mentioned, he takes an inclusive rate judged sufficient to cover the whole.

SYSTEMS OF DISTRIBUTION

Structural Features. These may be divided into the two classes of *overhead* and *underground*. The economic choice between them depends mainly upon the character of the area of supply, and the number and spacing of the prospective consumers. Generally an overhead system is more economical for widely scattered consumers, as in rural districts, where overhead distribution may be the only method commercially practicable. There is no sharply defined limit. In this country overhead distribution is handicapped by conditions not strictly economic; it is regarded as an objectional method requiring specific sanction from the Ministry of Transport as well as the local authorities in each case. In some other countries there is less difficulty both from the authorities and from landowners. It is of interest that in some countries, old laws which permitted easements over private property for access roads, irrigation channels, etc., to serve other properties, have been extended to electric lines.

Of late years there has been some relaxation of opposition, and of structural requirements in Great Britain, in recognition of the fact that the supply of electrical energy is a benefit which may be denied to many people where overhead lines are disallowed.

Some reductions in the cost of laying underground lines across agricultural land and along country roads are under trial; they promise to narrow the difference of cost between underground and overhead lines. That overhead lines are more liable to interruption by weather conditions is a factor to be considered. Overhead distribution will be dealt with in a later chapter. Long extra-high-pressure transmission lines, like those of the Central Electricity Board, are in a category rather outside the scope of this book.

Underground Systems. The necessary opening up of streets and the subsequent making good constitutes a substantial part of the total cost of installation. It is not entirely independent of the size and number of the cables laid, but for any route there is some minimum cost which cannot be reduced. It

depends mainly upon the kind of paving or road surface which has to be broken up and made good. The road authority has the option of doing the reinstatement itself, and usually has a schedule of rates from which an approximate preliminary estimate of the amount of this item can be made.

The cheapest method of laying cables underground is to bury armoured cables directly in the earth. An alternative is the laying of ducts into which cables are subsequently drawn. Cables to be drawn into ducts need not be armoured, but the resulting saving in cost will not suffice to pay for the ducts. The facility of drawing out and replacing a cable which has become overloaded or defective is not of much value as regards distributing mains which have service boxes at frequent intervals; cables of that class rarely fail. There are cheaper ways of protecting them from the pick-axe and the pneumatic drill than ducts or pipes. On the other hand, on routes of feeders leading out of generating stations, "Grid" substations, etc., to distributing substations, it is well worth while to consider the use of ducts. A common practice on feeder routes is to lay an armoured cable for the immediate prospective load (immediate means within five years or so) and one or more ducts alongside so that additional cables may be drawn in as required in the future. The author is inclined to advise the laying of high-tension feeders in ducts always, because of the facility of replacing a faulty or inadequate cable in minimum time and little or no expense for breaking up streets. This is open to discussion. It may be said that high-tension cables are now so reliable that the contingency of failures necessitating replacement is too remote to justify the expense of ducts. Perhaps so; yet it is easy to point out some rather considerable lengths of high-tension feeders which have been abandoned because failures had become too frequent and they were not worth the expense of recovery. Had they been in ducts there would have been some salvage value, and the ducts would have been available for new cables.

In the United States duct construction is almost universal, even for distributing cables.

In large towns the frequency of earlier underground structures which have to be dodged may be a serious obstacle to the laying of ducts. Gas and water mains, telephone ducts, sewers, subways, often leave little room and few straight runs of any length. Wherever an important line of feeder cables

has to be constructed in a town, it is essential to obtain as much information as possible about these earlier comers before deciding upon a route. The road authority should have plans of all underground works. They are likely to be incomplete as regards mains laid perhaps fifty years ago, and are rarely sufficiently detailed and on a scale to show exactly where a run is available. It is always advisable to make exploratory holes at road crossings and junctions, and in the vicinity of old buildings. Forgotten cellars and old foundations are not uncommon finds; it is better to discover them in advance than to have work held up by surprises. In some parts of London practicable routes have only been found many feet below the surface. Where a large number of feeders has to be taken from or to a generating or substation, it may be necessary to tunnel. Navigable rivers, canals, docks, etc., may entail tunnelling. It is eminently desirable to divide such large numbers of feeders into several groups leaving the station by as many ways. When tens of thousands of kilovolt-amperes have to be disposed of, the problem becomes serious. The temperature rise due to a number of heavily loaded cables within a restricted space has to be considered; cable tunnels have to be ventilated. This matter of heating by the bunching of cables has sometimes been overlooked, with serious consequences. Some years ago a power scheme was put before Parliament which included a cable tunnel which would have acquired a temperature over boiling-point under the estimated loads. The promoters were quite surprised when the fact was stated by an opposing witness, but had to admit the validity of the criticism. Such an oversight is perhaps not likely nowadays; there is such a considerable body of experience and research about the heating of cables in close proximity under various conditions of laying, soil, etc., that there is no excuse for ignorance. The British Electrical Research Association has published several reports on the subject of the heat conductivity of soils, which should be consulted.

Where the temperature of the atmosphere has large seasonal variations, appreciable effects are produced on cables laid near the surface. American data on this subject have been recorded and published for places like New York, Chicago, etc., where the seasonal temperatures range from nearly Arctic to quite tropical.

Duct materials include stoneware (a favourite in Great Britain); concrete blocks, and fibre (both largely used in the

United States). Stoneware and concrete blocks are practically indestructible excepting by violence. At one time cast-iron pipes were frequently used in this country; they are cheap and durable, and just why they have fallen out of favour is not very clear. They are better heat conductors than stoneware, concrete, or fibre. Their relatively high electrical conductivity seems to be, on balance, an advantage. It is better to have fault currents shunted from cable sheaths by such a low resistance path than to have them confined to the sheaths. Sheaths may be severed by arcing, and a broken sheath on a faulty cable in an insulating duct is highly objectionable. An iron pipe is a protection from stray traction currents which attack cable sheaths. Stoneware and concrete block ductings provide a larger number of ways within a given cross-section than pipes; for more than four ways the stoneware ducting is probably the cheapest in Great Britain, as the large demand of the Post Office has induced large scale manufacture on economical lines. For such special locations as on bridges where the cable way is along or under girders, etc., iron or steel pipes may be the only practicable protection. The circumvention of obstacles by a small degree of curvature is more readily arranged with pipes than with the block type of ducting. Curvature is to be avoided as much as possible to facilitate drawing in cables, but a little may be allowed with discretion. In mining districts subject to subsidence, pipes have withstood the effects better than block ducts.

This question of ducts, cable-ways, etc., is of importance in connection with feeders. Distributing cables cannot be deeply buried; they must follow the building lines fairly closely, and must be easily accessible for jointing services. The proper place for them is under the footpaths. Little is to be gained by using ducts; but it is necessary to give the cables some protection. Concrete slabs, creosoted planks, and bricks are frequently used; probably the creosoted plank is the most effective, for the "feel" of a pick striking a plank is very distinctive, and a long plank is less easily shifted than a slab or brick. It may be a wise precaution to pre-empt a path for a feeder alongside a distributing cable by laying a duct at the same time.

Electrical Features. Since the object of a distributing system is to supply energy in a form suitable for use by consumers, and since lighting is one of the most general uses, it

may be said that distribution voltages have been determined by lamp voltages. The earliest distributing systems of the early 1880's were at 100 volts or a little over, because that was the highest voltage for which filament lamps were then commercial articles. Edison adopted 110 volts for his first public supply stations. The odd 10 volts was a provision for voltage drop in the mains—the origin of the multiples of that number which figure in modern standard voltages. In the United States the majority of undertakings still work at 110 to 125 delivery voltage to consumers. Delivery voltage is a very important factor in the cost of a distributing system since the current corresponding to a given load is inversely proportional to the voltage. Comparing, say, 100 and 200 volts, the voltage drop along a given conductor for a given load will be twice as great at the lower as at the higher voltage; the percentage drop and the loss will be four times as great. Doubling the voltage, quadruples the load which can be delivered with equal efficiency. These considerations produced a pressure upon lamp makers to make lamps for higher voltages, leading to the 200–230 volts now prevalent in this country.

Before the incandescent lamp was a commercial article, the "Series Constant Current" system was developed and in somewhat extensive use for the supply of arc lamps. Several methods of maintaining the output of a dynamo at a constant current value, the dynamo voltage varying with the number of lamps in use, were developed in advance of the constant voltage variable current dynamo for parallel distribution. Dynamos for from 10 to 60 arc lamps in series, giving currents of the order of 6 to 10 amperes, made possible the distribution of arc lamps over very long circuits. The voltage drop in the lines did not affect the lamps; it might mean that a few lamps had to be sacrificed from the nominal ratings of the machine. The system was mainly used for street lighting, and survives to a considerable extent in the United States for that purpose, though rarely, if at all, with arc lighting dynamos. Arc lamps and sometimes groups of incandescent lamps dividing the current were installed in shops and other private establishments. It could not serve for a general public supply. It is not really practicable to install wiring to take the full load current at every consuming point in a house; nor particularly safe to have voltages of the order of 1 000 or more volts to earth on consumers' wiring.

At an early date high-tension d.c. series systems were used for transmission in a few British towns. The series circuit fed the motors of motor-generator sets in substations. The generators at 100 volts or thereabouts fed l.t. parallel distributors. There were batteries in the substations. During low load periods some of the motor-generator sets could be switched out, and the voltage or number of the generators in the main stations reduced. This system greatly increased the economic distance range of supply over that possible with direct generation at consumers' voltage. The substations were necessarily expensive; the system could not compete with a.c. transmission, but one or two examples survived into the 1920's. Particulars of the regulating arrangements, etc., are now only of historical interest. They may be found in the contemporary electrical journals of the 1880's, under the titles of "Oxford," "Wolverhampton," etc., systems.

There are some advantages in constant current series working for long distance transmission; one such system has been in successful use in France for many years.

The incandescent lamp called for a parallel system, so that any lamp could be turned off or on without affecting any other, i.e. the voltage at the consumers' terminals had to be substantially constant irrespective of the consumers' load. As above mentioned, 100 volts was at first the lamp limit; the first supply mains at that pressure were two-wire. This was soon developed into a three-wire system, in which the load was divided between the positive and negative "outers" and all loads connected to the middle or neutral wire. This was a series-parallel system; it halved the voltage drop for a given current in the outers, or doubled their load capacity as regards voltage drop. As each outer could carry double the current, the capacity of the pair was quadrupled at the expense of providing a third conductor. The middle or neutral conductor was necessary to carry any out-of-balance current due to unequal loading of the outers. Unequal loading produced different voltages on the more and less loaded sides; this was taken care of by balancers and regulators of several kinds in the station. The three-wire system was immediately successful, and was adopted on a great scale. It is almost universal to-day with both continuous and single-phase current distribution. An extension of the three-wire system to a five-wire one was made by Dr. Hopkinson, and adopted in a few places. That

had a more pronounced "series" character; it economized in conductor capacity and largely extended the practicable range of distribution. It had some considerable disadvantages; the voltage to earth of consumers on one of the outers was double the service voltage; balancing was more troublesome. The commercial production of 200-volt lamps put the five-wire system off the map, because the simpler three-wire system had its capacity multiplied by the change of voltage, at less cost.

Distribution by alternating current and transformers was developed almost as early as the constant voltage continuous current system. Gaulard and Gibbs installed the first of that kind in this country. They used a constant current series circuit with all the transformer primary windings in series with the line. This was not satisfactory for variable loads in parallel on the secondary sides. Ferranti, in this country, saw that a parallel connection on both sides would meet the requirements; also that the transformer, being a static machine, could be wound for a very high voltage on the primary side, and for any desired voltage on the secondary or consumers' side. The outstanding advantage of alternating current systems, considered solely from the point of view of economy of transmission, is that the transmitted and distributed voltages can each be selected to suit the economic and utilization requirement without the intervention of moving machinery, which is indispensable to voltage transformers on continuous current systems. The early alternating systems were single-phase, mostly with transformers on each consumer's premises. That proved wasteful and otherwise objectionable. Sub-stations with large transformers feeding distributing mains have supplanted the individual transformers for general supply. The three-wire system of distribution was appropriated from continuous current practice. Three-phase generation has displaced single-phase for reasons quite distinct from those connected with distribution, even where continuous current and single-phase distribution are in use.

Three-phase transmission and distribution are more economical than single-phase; just how much more economical depends upon the conditions of comparison. A three-phase line may be regarded as the equivalent of three single-phase pairs of conductors of twice the individual section, delivering an equal load with equal losses; the three conductors of the smaller section taking the place of the six of larger. The ratio is then

one-quarter of the conductor weight for the same duty. This takes no account of voltage difference and is for the most favourable re-arrangement of the supposed three single-phase windings or alternators, viz. in star and for a balanced load. If conditions as to potential difference between the conductors or to earth are to be met, or if a fourth neutral conductor connected to the star point is needed for out-of-balance current, the gain will be less.

Three-phase transmission lines are of three conductors. In this country the neutral or star-point of the system is earthed at the source; the three conductors then have equal and symmetrical voltages to earth. Unearthed neutrals are in favour on the Continent; the arguments pro and con need not be discussed here. The usual high-tension voltages for feeders and interconnectors within the area of supply of separate undertakings in this country are 6·6 and 11 kV. There are some instances of 22, 33, and a few of 66 kV. among the larger undertakings.

Both continuous ("direct") current and single-phase generation must be considered as obsolete for public supply on any but the most limited scale. The primary feeder system will always be three-phase.

In large towns there is a tendency to employ a system of primary mains at 33 kV. supplying substations where the voltage is stepped down to 6·6 or 11 kV. on a secondary system of feeders to the substations supplying the ultimate distribution system. In some of these places the higher voltage was no part of the original design, but became necessary in consequence of the growth of load, and/or the difficulty of getting sufficient cable-way capacity on the routes leading from the generating stations. The lower voltage lines of the C.E.B. are at 33 kV.; it is a common voltage for interconnecting lines between grouped generating stations and is likely to become usual for primary distribution.

The standard system for detailed distribution is three-phase four-wire. In Great Britain the standard voltages are 230 volts phase-to-neutral, 400 volts between phases, or outer conductors. In the United States (and some continental countries) about half those voltages are more general. In most countries some standard voltages are prescribed by regulations. There can rarely be good reasons for adopting any other than the three-phase four-wire distribution in new work. There is a

great extent of single-phase distributors surviving from earlier conditions. To-day these are mostly fed from three-phase substations; the several distributors being grouped to get approximately balanced phase loads. It may be expected that these will be gradually converted to four-wire systems.

There are also some surviving continuous current three-wire systems fed from converter substations. The conversion of these to the standard system presents somewhat serious problems, which will be discussed in their place.

There are some purposes for which the standard system is not applicable. The most important is that of electric traction, tramways, trolley buses, and railways. These require continuous current at voltages of from 550 to 3 000. In the past it has been common to provide generating stations for these undertakings. A good many survive. In any case the traction supply has to be separate from the general distribution; hence where both general and traction supplies originate in a station used for both, substations with converting machinery are put down for the traction supply. The independent traction station will cease to be usual. The Grid supply at 33 kV. to traction substations will become increasingly common for railways.

These and other special cases will have to be discussed separately.

The foregoing discussion has been somewhat wide and general; it is desirable to consider in more detail the particular features of different systems of distribution. Overhead lines must be given a separate chapter, but some of the electrical features are common to both overhead and underground lay-outs.

Series Systems. In order of priority the series system stands first. Both alternating and continuous current were used in the late 1870's with arc lamps, mostly for street, railway station, and works lighting. The alternating current systems included the Jablockhoff (candle) and German Siemens, both rather extensively used in Europe. In the United States, Brush and Elihu Thomson invented continous current dynamos and lamps, together with regulators, etc., to work with them. The essential feature of the series system is that the current is uniform all round the circuit. The dynamos and regulators were designed to that end; lamps could be switched on and off independently; and the dynamo voltage was adjusted to suit the number on circuit, sometimes by alteration of the engine speed. As such dynamos were made for as many as 60 arcs in series, needing

about 3 000 volts at the terminals; they were the pioneer high-tension machines. As the performance of the lamps depended only on the constancy of the current, voltage drop in the mains was of no other consequence than that it might reduce the rating of the machine in lamps. For the purpose of running lamps distributed over a large area, several miles of streets, etc., this system had good points. The current being usually under 10 amperes, only a small conductor was required. A No. 8 S.W.G. was commonly used for overhead lines; the line was essentially a loop of one conductor which might take any complicated course to pick up lamps over distances of several miles from the station. It was the first method by which electrical energy was commercially distributed over quite long distances; the most distant lamp ran as well as the nearest. Evidently the losses in the line were constant so long as the current flowed; that was not a serious matter for street lighting and other uses where either all the lights or none were required at different times.

The series constant current system survives extensively for street lighting in the United States, but has been generally converted to alternating current supplied from constant current transformers; the lamps are now mostly incandescent, either taking the line current direct or fed by individual transformers with the primaries in series. Some trials of the new gaseous discharge lamps on series circuits are in progress.

It seems that the series system is worthy of consideration for street lighting; the lighting can be controlled from the supply centre without switching circuits or time switches, independently of the general distribution. In the early years referred to there were cases of supply to private consumers: both arc lamps and groups of incandescent lamps were put into shops, hotels, and private houses. But this was a transient condition; the necessity of taking the whole current to every consuming device and the existence of perhaps over 1 000 volts to earth on consumers' wiring were inconvenient features. Nevertheless, good pioneer work was done before any other means of distributing to long distances from the source were available. It has already been noticed that the first a.c. system with transformers was a series system as regards the transformer primaries.

A high-tension continuous current series system has been in use in France for many years quite successfully; for very

long distances continuous current has some advantages over alternating current, and there is a distinct possibility that it may be so used in the not very distant future. This, however, is outside the scope or examination of distribution systems.

Parallel Systems. These were developed to serve incandescent lamps. An ideal parallel system maintains a constant voltage at each point of use irrespective of the load there, so that lamps can be switched on and off independently. In practice there must be some variation of voltage with the load on the mains. The general problem of parallel systems is to keep the variation at every point of use within tolerable limits and at the same time the cost of the mains within commercial limits. It is unnecessary to recount here the stages of development of dynamos and regulators aiming at the maintenance of a steady voltage over a distributing system. The relation between economical distribution and lamp voltage has already been noticed. The outcome as regards distribution was the three-wire system; ultimately for 200–230 volts (exceptionally, 250 volts) with double the lamp voltage across the outers of continuous current mains. The radius of economical distribution at that voltage is small. In large cities it involves generating stations near the load centres, with considerable difficulty in finding sites at reasonable prices—in coaling, ash removal, and especially with circulating water. That practice, once prevalent, is obsolete. Direct generation of continuous current is good only for small areas and loads. A radius of perhaps 2 miles may be covered economically, but if the load in the area justifies generating units of more than a few hundred kilovolt-amperes, alternating current generation will usually be more economical.

Excepting in such small area cases the three-wire continuous current system only survives the abandonment of the in-town generating stations, and is now fed by rotary converters, motor-generators, and mercury arc rectifiers supplied through three-phase high-tension feeders from large stations. No engineer would plan such a combination to-day.

Alternating current parallel systems were at first single-phase. The advantage of independence of the primary and secondary voltages of transformers allowed the use of high voltages between the generating station and the loads; the ultimate distribution could be kept within economical distances from substations; and over large areas the savings on

the distribution were great, as the generating stations could be placed on cheap sites, with condensing water, coaling and ash disposal facilities. In point of time alternating current and continuous current general supply systems developed together; and there was a lively controversy between the advocates of the two which is now of only historical interest.

So long as the chief use of electrical energy by consumers was for lighting, there was little to choose between a.c. and d.c. The a.c. systems adopted the three-wire distribution and similar voltages to the d.c. systems; but the change from the original 100/100 to the higher range made possible by lamp improvements was considerably more tardy than on d.c. systems because the cost and capacity of the l.t. distributors was of less concern. An increase of capacity could be obtained by putting in more transformers at shorter intervals.

Where motive power was required by consumers, single-phase supply was at a disadvantage. Some fairly large under-takings introduced two-phase supply to meet demands for power. It was quite easy to run both two-phase and single-phase distribution from a common station and transformers.

Three-phase generation and transmission was originally utilized for power transmission. The history of its develop-ment is very interesting, though this is not the place to give it. The earliest examples on a large scale seem to have been for traction purposes, e.g. for the Dublin Tramways where three-phase energy was converted to continuous current by means of rotary converters in substations. It was well known that three-phase alternators were smaller and cheaper as well as more efficient than single-phase machines for like outputs; also that three-phase motors of good characteristics could be made more cheaply than d.c. motors. The economy of three-phase transmission and the success of the tramway examples of converting substations led to similar installations replacing the in-town stations of the d.c. systems in large cities; eventu-ally to three-phase generation as the most economical for large stations in general and three-phase transmission as the best means of inter-connecting a number of stations; and so to the "Grid" of the British C.E.B. and similar schemes in other countries. So it may be said that the continuous current undertakings led the way in three phase practice, but they did not at first change their distributing systems. Single-phase undertakings gradually adopted three-phase generation on

account of the superiority of three-phase generating plant, especially turbine alternators. Alteration of their mains to the three-phase form involved considerable cost. In some cases they put in converter stations and distributed continuous current in parts of their areas where there was a considerable demand for power.

The superiority of the three-phase four-wire distributor over any competitor is now universally acknowledged. The generalization of three-phase bulk supply, and the official standardization of the four-wire system make those plans practically universal for new work in this country and most others, though continuous current and single-phase distribution are likely to survive in established undertakings for a time.

In addition to traction, certain industries require large continuous current power. Such are aluminium manufacture, copper refining, electro-plating, chemical works, and some metallurgical furnaces. If these are supplied from general mains some kind of converter is indicated. Even where such industries have their own generating plant, it will usually be three-phase at one of the standard voltages.

CHAPTER V

PRIMARY AND ULTIMATE DISTRIBUTION

FOR reasons adduced in the preceding chapter, it may be assumed that in any new undertaking there will be a "primary" system of high-tension feeders, and a "secondary" system of distributing mains. Also that existing undertakings will in time conform to this general plan. In some cases there may be three systems, a high-tension transmission to load centres ; a lower tension distribution to individual transformer substations, and the distribution mains.

The source may be a local generating station designed for the particular area only. In the near future the source will be either a "selected" station, feeding into the Grid as well as the local network, or a Grid substation. The station voltage will usually be 3 300, 6 600, or 11 000 volts. Either of these is suitable for the direct feeding of transformers. A Grid supply will usually be at 33 000 volts. That is not suitable for the direct feeding of small substations as both cables and transformers for that pressure are unduly costly in small capacities. It is a quite suitable pressure for a primary system feeding large substations of, say, 5 000 kVA. capacity each or over. Extension to smaller outputs now in progress may make this statement out of date before long.

The choice of voltage for the more detailed feeding lines to distributing substations is mainly determined by the radius of distribution ; voltage drop to the most remote substations may probably be the deciding factor. As explained in Chapter III, the most economical current density can be calculated from the Kelvin relation if the load characteristics can be forecast ; the cost of energy at the source and the price of the cable being also known. If the supply is on a bulk tariff with a kilowatt charge, the kilowatts of the losses at the maximum load should be brought in. Evidently a given voltage drop per mile is a smaller percentage for a higher than for a lower delivery voltage, so that the permissible maximum distance of transmission rises with the voltage, as does also the kilovolt-ampere capacity of a given size of conductor. On the other hand the permissible maximum current density for heating

falls off with increase of working voltage. There is not a very great difference in this respect as between, say, 6·6 and 11 kV. cables, nor in the cost per ton of copper for equal sizes.

Probably for such areas as those of a town of up to 250 000 inhabitants, and for a maximum feeding distance of about 7 miles, 11 kV. will be found economical; but it is always worth while to compare the calculated costs and charges for two or more pressures. If the available cable routes are constricted or difficult, or if a very large total power has to be got away from the source, it may be advisable to select a higher voltage in order to reduce the heating of the soil, subway, or tunnel where the feeders are bunched. This consideration may lead to the adoption of, for example, 33 kV. to get the load out, transforming down to 11 kV. at some convenient place. If the source is a Grid substation 33 kV. will be available there.

The ultimate distribution in this country will usually be on the standard three-phase four-wire system at 400 volts between phases and 230 volts between each phase and neutral. Services to small consumers will be single-phase, one phase wire to neutral. It is desirable to keep the phases balanced locally; providing for that purpose some means of transferring individual consumers from one phase to another as may be required. The ultimate distribution system is the seat of the largest and most expensive losses; the largest because the current is so much greater (27 times as much on a 400 volt distributor as on the 11 kV. feeder to a substation), the most expensive because the losses there have to be charged with the loss factor and the capital charges of all the preceding mains, transformers, etc. The voltage variation at consumers' terminals has to be considered, as that is likely to be the limiting factor in the loading and radius of distribution which can be allowed. The ultimate distribution system is the largest item in the cost of the whole, and therefore the design merits the closest attention. Unfortunately, it is the part least amenable to forecast of the loading conditions.

In dealing with existing single-phase distributions there will almost always be some three-wire distributors which it is not possible to change over to four-wire immediately; whether for reasons of the cost or merely because the conversion must take place by stages. Such three-wire distributors may be connected into a four-wire system through suitably wound transformers or auto-transformers, care being taken to balance

the loads between the phases as nearly as may be. Large consumers, especially those with much power demand, are most economically supplied at high-tension, with transformers on or immediately adjacent to their premises.

In densely loaded areas with many large consumers, the high-tension may become virtually the ultimate distributing system, with transformers for each large consumer. It is quite on the cards that this reversion to the old "house transformer" will become necessary, because of the sheer physical difficulty of finding room for the large low-tension mains otherwise necessary.

At the other extreme, in a rural area with farms, houses, and villages separated by considerable distances, a similar arrangement will be required. There may be some short low-tension distributors in the villages to individual consumers. The substations in these circumstances will frequently be pole-mounted transformers.

CHAPTER VI

NETWORK TYPES

THE types of distribution networks may be divided into—

(1) RADIAL. Feeders radiating from the supply source to individual loads or load centres, or to lines of loads strung along the feeders. This type is used for high-tension feeders in towns, each feeder supplying one or more substations; and also finds application in districts with scattered loads.

(2) RING. Ring mains are those surrounding a load area. They have rarely been part of the original lay-out, but have been superimposed upon the original radial or mesh arrangement as that became overloaded. Particular cases are those of large cities supplied from a number of generating stations. The ring mains interconnect the stations forming an extended bus-bar, usually at either the generating voltage or a stepped-up higher voltage such as 33 or 66 kV. Primary substations are located on the ring or on branches from it. This type of primary network is likely to become more common; it is worthy of consideration for comparatively small towns and for rural districts. The original radial lay-out has in many cases developed into a series of rings by the connection of what were dead ends into continuous loops. The ring type of feeder has the advantages that each point may be fed from two directions; and that individual load variations have smaller effect upon the voltage drop; the point of lowest voltage shifts with the distribution of loads around the ring in an advantageous manner.

(3) MESH. The mains form a network with loads distributed along the lengths between the intersections. The feeding point or points are located with as much regard to the local load distribution as may be practicable. In built-up areas the distributing mains inevitably take a mesh form conforming to the street lay-out. Every load point can be reached by two or more routes. The load variations are partly compensated for as regards voltage drop by varying division among different routes, and the conductors are more fully utilized by such division.

These three types merge into each other. Radial feeders become rings when their ends are connected together; if

occasion arises for cross connections they become meshes. One may regard a mesh network as a series of rings. It may be fed from the supply source through a number of radial feeders. This is the usual arrangement of d.c. distribution in towns. Feeders radiate from generating or converter stations to suitable points on a meshed network of distributing mains. In a.c. systems the feeders are usually high-tension to transformer substations.

The radial feeder lends itself readily to calculation. The simplest case for calculation is a point-to-point transmission of a certain load. If the load is distributed along the feeder instead of being concentrated at the end, the calculation is only increased by having to be made for each section between the loading points. Such a load distribution calls for a stepped-down copper section to give approximately uniform current density along the whole length. It may be the Kelvin density, or determined by the voltage drop at the maximum load. A radial feeder may reach some of its loads by branches off its topographical line; it then becomes a "tree." This introduces an additional element into the calculation, viz. the drop in the branches at their individual maxima loads. Two adjacent radial feeders with branches may become a ring by the meeting of branches from each, or by connecting across their ends. Such connections do not introduce serious difficulties into the calculations; the ring may be regarded as two feeders in parallel for normal conditions. But if the possibility of the ring being cut at some point and some of the load having to be fed the long way round is considered, the cable sizes will have to be heavier than are required for normal working in order to keep within either the permissible voltage drop or cable heating while the abnormal conditions prevail. As these will be of the emergency order, the extra copper losses are not of consequence; but heating and voltage drop must be regarded.

A ring feeder system gives some increase of reliability of service over the plain radial feeder plan, at the cost of extra capital charges.

In large town areas ring arrangements have often come into existence in the course of development as reinforcements of an original radial system, the radii or branches from them meeting at certain substations. They may be permanently connected at such points, or only connected on special occasions; frequently the substation load is split between the two. A

multiplicity of such interconnections becomes a mesh network.
The question of working the whole network interconnected or
divided into sections—usually with switching arrangements to
modify the degree of interconnection—is largely one of reli-
ability and convenience of operation. It has some relation to
the size of the generating or transforming units at the immediate
source. It is convenient to be able to group feeders to give a
suitable load to each unit or to each bus-bar section. There
may be maintenance of synchronism between the sections by
reactance connections. For several reasons it is desirable to
take precautions to prevent feeders on different bus-bar sections
being paralleled at substations. The feeder system is run as a
number of separate sections each supplying its own group of
substations; transfer of substations from one feeder section to
another is made possible by controlled switching.

A different plan is followed in some large American cities.
The high-tension sections are not paralleled at the generating
stations, but through the low-tension network, "paralleling
at the load."

If one could plan a large system for its ultimate development
(which has not been the case in the past, and may never be),
probably a radial system with some rings would be adopted.
Something like that is being developed in certain large cities
where the original high-tension system has become a meshed
network, and is now having superimposed upon it a feeder
system at a much higher voltage, 22, 33, or occasionally 66 kV.,
on a radial plan to large step-down stations. The original
primary network thus becomes a secondary distribution.

A primary high-tension ring has been adopted in some large
cities where several generating stations serve the whole area.
Paris is an example. A 60 kV. ring main surrounds the city;
it is fed into by a number of generating stations, and by
transmission lines from distant water-power plants. At suitable
places, having regard to the local load distributions, there are
stepping-down substations to the local secondary networks
which were originally primary networks. Similar ring main
systems are in course of development in Berlin and in parts of
the London area. But see a later note of the plans in prospect
for the Berlin area. In Chicago a 66 kV. ring connects all the
generating stations and transmission line termini, and is
treated and worked as a general bus-bar.

Where the growth of load makes such a superimposition

necessary there is an opportunity for planning from known data. The load distribution is known: its rate of growth can be forecast with fair accuracy.

There is some difference of opinion about the advantages of running all high-tension sections interconnected into a meshed network in parallel; or in separate sections. For maximum economy and minimum voltage variation due to local loads parallel working is the better. It may, however, concentrate very large amounts of energy on any fault and impose excessive duties upon circuit-breakers; hence separate working is preferred by many. A compromise method is the so-called *loose linking* plan. Sections are linked at certain points through lightly set circuit-breakers, and a fault on any section trips the circuit-breakers. Generally this is applied where each section is fed by a separate station, or off separate bus-bars. The tripping leaves the station feeding into the fault to deal with its own troubles, which it may be able to do in a fairly satisfactory way. At the worst the remainder of the system is unaffected.

From this discussion it will be seen that the design of a high-tension system for a large city has to take into account other than purely economic considerations, and merits very careful detailed study.

Feeders into rural districts with scattered loads—farms, small villages, perhaps factories—are necessarily at first of the radial type. They can be designed for maximum economy if the loads and load characteristics can be forecast. Estimates are rather speculative. Such lines will usually be overhead. Certain minimum conductor sizes are necessary for mechanical safety and are prescribed by regulations. Within limits, the cost of a line of posts or masts is little affected by conductor size; but at some point the limit is reached, and heavier and more costly supports are needed, so that the cost goes up by large steps. Rural feeders usually follow main roads: their remote ends may diverge widely. Reliability of supply will be improved by joining the ends of pairs into rings. In some cases the ring main is a natural way of picking up the various villages, etc. In any case it is worth while to make a close study of load distribution and line routes over a rural district to see whether it can be served by ring mains. The two-way feeding and the consequent minimization of the effects of the accidents to which overhead lines are liable is of much value.

In built-up areas the ultimate distributing network is always of the mesh type, conforming to the street plan. There are some essential differences between the elements of economic design for d.c. and a.c. networks. For d.c. distribution the substations are relatively expensive in first cost and attendance, small ones being more expensive for their capacity than large ones. Consequently, the general practice has been to set up fairly large substations feeding correspondingly large areas by radial feeders to suitable points. Economic planning involves a balance between the costs of fewer substations with longer feeders and more substations with shorter feeders. This position has been recently modified by the advent of the mercury arc rectifier which makes the establishment of more and smaller substations less onerous; but they remain more expensive than transformer substations of equal capacity. For a.c. distribution the relatively low cost of transformer substations favours direct feeding of the networks by substations at such points that the average length of the low-tension mains is small. In effect the low-tension feeders of the d.c. system are replaced by high-tension feeders and transformers. The effective capacity of the network can be multiplied by adding substations as necessary. Design conditions are more elastic and load growth can be met at less expense. The areas supplied from each substation are smaller than those which are economical with converting substations.

CHAPTER VII

TOWN DISTRIBUTION SYSTEMS

THE last chapter dealt in general terms with network types and their applications. It is desirable to discuss the planning of a distribution system in more detail.

In a built-up town it may be assumed that there will eventually be mains along every street: a plan of the ultimate development in outline will be a street plan of the town, in general a meshed network, with a variety of shapes and proportions of the meshes.

A preliminary examination of the relevant data (i.e. the cost of energy, the anticipated load and form factors of the loads, and the cost of cables in terms of the copper weights,) permits the determination of the most economical current densities, as explained in Chapter III, and therefore of the mean radius of distribution from each substation which will give a voltage drop within the prescribed limits. The most economical current density and rate of voltage drop may not be the same for all parts of the area: differences in load and form factors as between industrial, residential, shopping, and other distinguishable districts may have to be taken into account.

The radius of distribution so found de-limits the area to be supplied from each substation. The spacing of the substations should be such that no part of the area is at a greater distance from one or more of them. Applying this to a map of the area will divide it into a number of sub-areas with a substation central to each. With the substations spaced apart 1·732 times the radius of distribution, the sub-areas will be hexagons with sides of length equal to the radius. Spacing the substations 1·414 times the radius apart, the sub-areas become squares of that length of side. The hexagon plan requires the minimum number of substations.

Such a plan gives the ideal number and positions of substations over the area considered. It is rarely possible to realize the ideal; the ideal affords data for preliminary estimates and a useful "yardstick" for comparison with any practicable lay-out after consideration of the run of streets and other conditions. Of the practicable alternatives the one

which approximates most nearly to the ideal or symmetrical disposition should be preferred.

The next step is to estimate the loads to be provided for in each of the sub-areas. The estimates should prudently forecast future conditions, say for five years ahead. Assuming that the district has had no electric supply, the best guide is statistical information from similar areas interpreted in the light of close inspection of the place. Experience shows that under-estimates of the growth of load are more expensive than optimistic estimates. If the planning is being made for extensions of an existing undertaking, or for the conversion of a d.c. system to a.c., there will be much more information about the loading and the probable growth of load than in a virgin district.

The estimates will give a total load for each ideal sub-area, consequently of the required capacity of each of the sub-stations.

The next step is to consider where the real substations can be placed. The form of the street meshes will usually suggest positions at street junctions whence the distributors can lead out in several directions. If the area loads are in excess of the transformer capacity which can be accommodated in under-street boxes, or kiosks, sites may have to be found for buildings or outdoor substations. The cost of such sites as may be available will have to be considered; a complication which may considerably upset the ideal locations. On this account as well as on account of the magnitude of the expected load, it may prove better to divide the substation capacity between two or more places in some areas. On the other hand, it may prove better to supply two of the ideal sub-areas from a single substation, at least as a first stage. The immediate load expected in some of the areas may justify deferring the cabling and substation provision to a later stage of development; but it is wise to have the plans ready for that development and fitting into the complete scheme. It may be remarked that experience shows that canvassing for consumers before mains are laid ready to supply them gives little indication of the demand which will arise when it can be met at once. Also, in this country there are compulsory areas within which supply must be available within a certain time of the grant of the powers.

The position of a substation in respect to the area which it

serves is a factor in the lay-out of the distributors in that area, since the copper section of the mains leaving the station must be sufficient to carry the whole load without excessive voltage drop or heating. Hence the location of substations may entail a departure from the most economical copper section. The cable sizes to be used must therefore be calculated with reference to voltage drop. Here comes in an application of the "yardstick." The sizes, lengths, and cost of the mains from the ideal central position can be compared with those consequent upon any other position; and the difference of cost may then be considered as the cost of departure from the ideal. Evidently alternative positions can be compared for selection when the costs of the different sites come into the reckoning.

In the outer suburbs of large towns the loads may be chiefly distributed along certain main roads. There are also some towns, not all of them small, of the "backbone" type; stretched out along a coast line, a valley, or a main highway. In these cases the mesh reduces to a "tree" with short branches; the areas served by each substation are substantially linear and of small breadth. There may be networks of streets running off the main line at some points; whether they require substations off that line, or can be economically served from some along it, is a question of the magnitude and distances of the loads.

Such strung-out districts are easily planned. The theoretical spacing of substations along them is twice the voltage drop range at the selected current density. A high-tension feeder will run the whole length. The low-tension distributor size will be chosen with regard to the estimated load. Usually the assumption that the load on each length will be lumped at the centre will be safe. Since the high-tension feeder runs alongside, an increase of load can be readily met by putting in additional substations. Also, any large individual loads can be supplied by transformers on or near the consumers' premises. Underground chambers or kiosks will generally be the most economical form of substation; but in this country 75 kW. is the maximum capacity allowed in underground substations, so that expedient may not always be practicable. Street corners will be the best locations, so that low-tension feeders into side streets can be taken straight out. Such side branches may take a partial ring form, leaving the "backbone" line by one street and returning to it by another. This is a very

convenient way of dealing with any long narrow area; every part of the low-tension distributors will be fed from two substations, either of which may be switched out for cleaning, testing, etc., without interruption of supply. In the original lay-out of such an area some of the branches may run out to dead ends, it is well to keep in view the possibility that at some future time there will be a call for branches joining across the ends. This is especially likely where the area is in process of building up. The branches should therefore be of sufficient size to carry the loads likely to be added by the developments.

In its simplest form the high-tension feeder along a "backbone" will be of the dead-ended radial type, affording only a one-way feed. Provision for increasing the capacity may be made by laying a spare duct alongside, so that a second cable can be pulled in when necessary. Then alternate substations may be transferred to the new cable. With appropriate switching arrangements the whole line can be given the equivalent of a two-way feed.

For the case of an already built-up area requiring substations of large capacity, the positions will be largely determined by the cost of sites for buildings or enclosures; and with regard to the run of the streets so that the mains can be got away comfortably in several directions. The major consideration is that no substation shall have to supply beyond the economical radius. The practicable positions may well depart from the regular spacing shown on the load plotted plan, but that plan will still be useful as a basis for estimating the loads which can be most economically supplied from each substation and the capacities of each. It may well be that the actual areas and substations location bear little resemblance to the original plan.

It is true that at present load estimates in new areas are at best scientific guesses; and the deduced substation capacities can have no greater accuracy than the data upon which they are based. But the most economical spacing is more definitely determinable; and even if the immediate load does not warrant equipping the full number of stations, the necessary sites may be pre-empted in advance, and the mains lay-out arranged to make use of them without much alteration.

The meshed network fed from each substation will be normally all in parallel. It is usually advisable to provide means of isolating individual street lengths by means of links in junction boxes at street corners. Whether these links should

be fuses is debatable. At least the fuses—if used—should be reckoned as protecting the cables; they should only blow for a sustained load little short of that which will permanently damage the cable. American practice favours solid connection of each network. If a fault develops in a cable it is reckoned that it will burn itself clear, severing the conductors. But that is for the American 115–125 volts phase-to-neutral. With the British standard 230 volts such self-clearing cannot be relied on. Fortunately dead short circuits on modern low-tension cables are rare. But the navvy's pick or the pneumatic drill may intervene.

Another aspect of linking up arises at points where the networks normally fed from different substations meet. They may be kept disconnected with link boxes or pillars available for linking up in abnormal conditions, or connected by relatively light fuses. As a general rule, the linking on special occasions seems preferable. That localizes the effect of any trouble to the particular section on which it arose. Light fusing has a partial effect of the same kind; on the other hand, if it becomes necessary to feed a part of a network from other than the usual substation, the light fuses must be replaced by links before that can be done. If adjacent networks are on different high-tension feeders, they are best kept separate, as there are considerable objections to the indirect paralleling of high-tension feeders through low-tension networks, unless that system is deliberately adopted.

In densely loaded areas the question may arise whether to put substations at a closer spacing than that determined by the radius of supply set by economical loading of the low-tension cables. If it is a case of original lay-out, the question can be solved by putting the additional cost of two substations against that of one of the same total capacity plus the extra cost and losses in low-tension mains from the single station. More often the question arises as the result of the growth of load beyond that provided for originally.

In such cases there can be little doubt that the increased load should be met by a suitably placed second substation. The alternative of putting more transformers into the original station and running low-tension feeders to the heavily loaded part of the network will usually be more expensive and less economical than a second substation; that will reduce the losses as well as increase the capacity of the network.

Feeders to suitable points was the original d.c. practice, justifiable because of the high cost of converting substations; the much lower cost of transforming substations completely alters the balance of economy.

It is just as true for "built-up" areas as for the "backbone" areas previously discussed, that it is more economical to add transformers at the right places than to reinforce an existing low-tension network. If heavy loads are imposed by new large consumers, it is always worth while to consider putting down transformers for such loads.

All this discussion is necessarily general. It is impossible to do more than point out the main considerations; every case must be considered by itself. The cost of low-tension networks and of the losses in them are such large factors in distribution costs that thorough consideration is essential to the planning of a soundly economic lay-out. The losses in the low-tension distribution are those of the highest cost (excepting those from under registration of consumers' meters), they include the capital charges and losses pertaining to the high-tension feeders and the transformers in addition to the generating costs.

The problem of economical distribution may be approached from a different angle than the one indicated in the foregoing discussion. In that the radius of distribution has been taken as that which will give the permissible voltage variation with the most economical current loading of the distributors run at the declared voltage at consumers' terminals. That radius may not be the most economical. Evidently the shorter the mean distance of transmission at the low voltage, the lower will be the losses. Reducing the mean radius means increasing the number of transformer stations, decreasing their individual capacity, and similarly decreasing the sizes of the low-tension mains. With sufficient data it is possible to arrive at a balance between the increased cost of the substations and high-tension feeders and the reduced cost and losses in the low-tension mains. In the limit there would be a transformer on the premises of every consumer; a return to the old "house-to-house" system of the early a.c. undertakings. If any consumer's load equals the capacity of the most economical transformer as arrived at by the balancing of costs suggested, it would probably be the least costly way of serving him. More generally the calculation would give the best size of substations and their spacing for an area of given load density. There is already

evident a tendency to proceed on the plan of laying low-tension mains of some one or two standard sizes and adding transformer stations at shorter intervals as the load grows, especially in residential areas. This too is not novel; it was done in a number of towns forty years ago, usually in substitution of the house-to-house system, which incurred heavy magnetizing losses, especially as the loads were mainly lighting with load factors of 10 per cent or less.

This problem of the most economical capacity of substations and the consequent sizes of low-tension mains will be treated more fully in another chapter.

From whatever bases used, the planning will show the selected position of the substations, and so the points which have to be reached by the high-tension mains. The position of the source of supply will usually be known; it may be one or more generating stations or a substation on an e.h.t. transmission line. The routes to pick up the positions plotted on a plan of the area may be more or less obvious. The h.t. mains may be planned as a number of radial feeders, as one or more rings or loops, or as a meshed network. It is highly desirable that every substation shall be fed, or be capable of being fed, by at least two routes. Generally, this can be done by looped radial feeders, or elongated rings, if there is only one source of supply. In some cases a ring around the area with radial loops to substations is an attractive proposition, especially around comparatively small towns. Evidently if the number of substations is large and their spacing small, the h.t. mains tend to take a mesh form.

The most economical h.t. voltage for feeding the substations has to be settled. As was observed in examining the economic efficiency of cables, the "cost per ton of copper" runs up sharply with the working voltage; that makes the economical current density higher than in lower priced cables. But high voltage cables cannot be run at such high current densities as those for lower voltages. To keep the working temperature within safe limits, the current density must be lower. One reason is that heating due to dielectric losses becomes important; a loss which increases with temperature. There is an extensive literature on this subject, and progress is being made in the design of cables for ever-increasing voltages. There is ample information about the safe loading of cables up to 33 kV., which is as high as one is likely to go in a distributing system.

The upshot is that the safe current loading of cables for over, say, 11 kV. is usually lower than would be given by an application of the Kelvin relation.

In practice the choice is limited to one of the standardized voltages, viz. 6·6, 11, 22, and 33 kV. At present there is a wide gap in the "cost per ton of copper" between 11 kV. and 33 kV., which may rule out the higher voltage for distribution purposes. A rough comparison can be made on the "cost of copper" basis. The same copper section worked at the same current density will deliver three times as much energy at 33 kV. as at 11 kV. If the respective prices were as three to one, the capital charges per mile would be equal. That comparison must be qualified by the fact that the 33 kV. cables cannot be worked safely at as high a density as the 11 kV. cables. A more practical comparison is between two cables to deliver the same amounts of energy. For example, a 0·25 in.^{2} three-core armoured cable laid direct in the ground for 11 kV. has a safe current-carrying capacity of about 363 amperes per phase; say 6 900 kVA. A comparable standard size of three-core 33 kV. cable has conductors of 0·15 in.^{2} the safe current is about 216 amperes per phase, or a loading of about 12 360 kVA.; say 1·8 times that of the 11 kV. cable. The relative prices per mile are at present in the neighbourhood of 2 to 1; so that on this basis two 0·25 in.^{2} 11 kV. cables of 13 800 kVA. capacity cost about as much as one 0·15 in.^{2} 33 kV. cable of 12 360 kVA. capacity. That comparison does not answer the question, which will give the lower total annual costs? The answer involves calculations of the load and form factors of the annual load; and the cost of the losses. It can be said that for a cost of 0·25d. per kWh. for the losses, at an annual load factor of 100 per cent, the economical current in the 11 kV. cables is much below, and in the 33 kV. cable decidedly above, the safe currents. The 11 kV. cables will, at this load factor, give higher total annual costs per kVA. delivered than the single 33 kV. cable: the higher losses more than counterbalance the lower capital charges. At some lower load and form factors the order will be reversed. It may be observed that the current density is the safe maximum for the 33 kV. cable, but the 11 kV. cables have a reserve carrying capacity much above the economical density, and can be used safely on a more peaky load.

There are other considerations than the transmission costs.

The 11 kV. cables will occupy more trench space than the 33 kV. cables of the same kilovolt-ampere capacity; they must be laid at some distance apart. At 12 in. centres the safe loading is 88 per cent of that of a single cable. If the cables are to be drawn into ducts, or carried on racks in tunnels or subways, the space occupation becomes an important factor in the capital cost.

Further, the cost of switchgear and transformers is higher at the higher voltage; the smallest economical size of transformer for 33 kV. primary is considerably larger than that of the smallest economical size of transformer for 11 kV. primary. This difference is in course of reduction.

It is probable that 11 kV. will be found to be as high as it is wise to go for a primary voltage for distributing substations if many of them will be of comparatively small capacity.

The problem of the h.t. planning is to settle routes which will pick up all the substations with a minimum of cable length and of street work, while giving a reasonable amount of alternative ways of feeding each one. It is worth while to spend some time and trouble in selecting the routes, and getting the maximum amount of information about existing underground works. Getting the whole lot of feeders away from the source may give some trouble, especially if that source is on the periphery of the area so that there is considerable bunching at the outset. It is advisable to spread them at the earliest opportunity.

As regards the general lay-out, a radial plan seems best on the whole. It may readily become a number of loops or rings, either at first or subsequently. A plain ring with loops reaching substations inside it will suit towns covering perhaps up to four square miles; of course, outlying areas can be served by outward going loops. Any densely loaded area is conveniently served by a ring around it.

The general idea is to break up the area into plots or strips each fed by a ring or a pair of radial feeders taking different street routes on the two legs and looped at the remote ends. This plan of breaking up the area into separate plots makes for convenience and reliability, and facilitates the measurement of output and distribution losses for each, an important matter. Some of the plots may have loads which justify the laying of two parallel feeders.

When a system is being planned *de novo* (as is assumed in this discussion), the original lay-out should be planned to allow

for future reinforcement of the h.t. system at the least cost, e.g. spare ducts laid along the routes, feeders at first dead-ended located with an eye to future completion into loops, etc.

It can hardly be advisable to plan a h.t. system as a mesh network. The advantages of a meshed system may be largely attained by running loops from two different feeders or rings into certain substations, where they may be linked in case of need. The l.t. networks from substations on different feeders will meet at a number of points. It is advisable not to connect the l.t. lines at such points. If the substations are on the same feeder, there is not so much objection. So much for the planning of a system *de novo*. The opportunities for doing that in towns of any size are now rare; but the principles may find application in modernizing existing undertakings.

The problems which arise in large towns are now usually (*a*) extensions to outlying areas of small dwellings; and (*b*) conversions from d.c. to a.c. or from non-standard to standard distribution. Both may have to be dealt with at the same time. The preliminary estimating of the loads and their distribution will have better bases than in a new area. Much of the load is in existence and there will rarely be much difficulty in arriving at a fairly accurate forecast of the rate of growth. The most debatable question may well be, how and to what extent the existing mains can be used for the standard system?

Where the continuous current mains are in the form of a three-wire meshed network fed at intervals by heavy feeders from converter substations (sometimes from generating stations) and single conductor cables with the middle conductor of smaller section than the outers, they are not suitable for use in a four-wire three-phase system.

The points where the l.t. feeders connect into the network indicate suitable positions for transformer substations if the necessary site accommodation can be arranged there. The converting substations will be served by h.t. feeders and can be used as centres from which radiate the sub-feeders to the transformer stations. In most cases it will probably appear that more transformer stations than indicated by the old l.t. feeder points will be advantageous.

If the h.t. supply to the converting stations is at anything over 11 kV., it will probably be best to step down the voltage at those stations to one more suitable for small transformers. The best locations and capacities for the transformers over the

whole area previously served can then be planned as if for a new area. These locations may or may not include the old feeding points. That is mostly a question of the cost of available sites, but should also take into account the known distribution of the load. If there are spare ducts or ways from the old converting stations available for the h.t. sub-feeders they should be used as far as possible. As each transformer station is connected and equipped, the substitution of four-conductor distributors for the three-conductor mains can be commenced. The substitution will be much facilitated if the old mains are in ducts, or spare ducts are available along their course. Evidently, if ducts occupied by the old mains are to be used, the old mains must be drawn out first after cutting all the service connections on each length. To minimize the extent and duration of the interruptions to supply is a problem calling for the greatest ingenuity. In some cases it has been solved by laying temporary mains on the surface of the footpath and bringing the services up to them while the new distributor is being laid and service boxes placed upon it. If the old distributors are laid directly in the ground, the new ones can be laid alongside with service boxes ready and the services changed over progressively.

As the old mains have considerable value as scrap copper one object of the change-over plan should be to salve as much as possible of them. There may be some large consumers who can be economically served by transformers on their premises, or near them; they may be dealt with in that way, at a convenient stage of the proceedings, with advantage.

A less radical plan is to retain the old d.c. distributors but to feed them at more frequent intervals from mercury rectifier substations. That is more practicable in relatively lightly loaded areas, such as those of the suburban residential type, than in densely loaded areas.

In some cities having central areas with heavily loaded d.c. three-wire networks, the idea of converting them as a whole seems to have been indefinitely postponed; meantime a four-wire a.c. system is being laid alongside to serve all new consumers and to take over the old services as opportunity offers by change of tenancy or otherwise. That may be economical up to a point where the reduced load on the converting stations makes their commercial efficiency very small, when the problem of complete change-over will have been much eased.

Where the existing distribution is single-phase—usually three-wire—the simplest plan for immediate application is to divide the distributors leaving each substation among the three phases to balance the phase loads as nearly as possible. If three-phase transformers are used, there will be some internal balancing as regards the h.t. feeders. The transformers should have two secondary windings per phase with all the ends brought out; then they can be connected into a virtually six-phase system on the l.t. side. When four-wire distributors are eventually substituted, the same transformers can be used for the four-wire distribution. This is on the assumption that the three-wire system is 200 volts on each side of the neutral, and that the four-wire system will be on the standard 230 volts on each phase wire to neutral. Each pair of 200-volt secondaries connected in series will then give the 400 volts for one phase. It will not do to connect the single-phase neutrals to the neutral of a four-wire system for obvious reasons.

It is possible to convert single-phase three-wire mains into four-wire mains by running alongside a single-core cable as a neutral. This means using the old neutral of the three-wire mains as a phase wire; generally that conductor is of smaller section than the outers so that the phase voltage drop on that phase will be greater than in the other two. Also the single-core neutral will have greater losses than if it formed part of a four-wire cable. In lightly loaded areas this plan may be worth while, but it needs careful consideration of all the factors; it may be regarded as a temporary stage to defer expenditure for a time.

In every case, whether a new system or the conversion of an old one is being planned, there are considerable advantages in deciding to use only a few standard sizes of distributing cables, and of transformers. The transformer sizes may usefully include the largest which can be placed in under-street chambers and kiosks.

A good description of a planned system for a large city, was given in a paper by Mr. M. W. Humphrey-Davies, on the "Electric Supply System in Berlin," published in the *Journal of the Institution of Electrical Engineers*, Vol. 80 (1937), p. 305. It is an unusually full account of the planning for economy and reliability of supply to a large city, and of the reasons for the methods devised.

At the time of the author's examination, the Berlin system

was in process of conversion from a number of systems of different dates and kinds to a general plan. That plan is one of a primary system of 30 kV. mains and a distribution system at 380–220 volts. In the urban districts the 30 kV. mains will feed transforming substations on 380–220 volts distributing grids. Each grid will cover an area of from 1 to 2 square miles; and be supplied through at least six 30 kV. feeders each of 0·15 in.^{2} copper (or aluminium equal), rated at 6 000 kVA., capable of feeding 15 transformers each of 400 kVA. Each feeder will serve one transformer only in any substation. The feeders are "radial" but from several centres, generating stations. The transformers on a feeder are tripped on the l.t. side whenever the feeder is tripped out. The protective system which effects this, operates through pilot wires and looks rather elaborate. There is some resemblance to the "paralleling at the load" of some American systems. This "two-voltage system" is in course of development to supplement and ultimately displace the "three-voltage system" where there is an intermediate 6 kV. distribution stage between the 30 kV. and the l.t. mains. It has been adopted as more economical where the load density exceeds 2 800 kW. per square mile.

In the suburban districts with lower load densities the three-voltage system will be retained and extended, the 30 kV. feeders serving 30/6 kV. substations, and the 6 kV. mains serving transformers to the distribution voltage.

The 6 kV. system in the urban areas will ultimately be confined to the supply to large consumers, and to small areas around generating stations where that voltage is used for auxiliaries.

The two-voltage system has proved very satisfactory during some years of experimental development, which includes the working out by trial of sundry switchgear and protective plans.

It is intended at some future time to run 100 kV. ring mains around Berlin; there are already 100 kV. lines from distant stations feeding the system at two points. Further, it is intended when the peak load reaches 1 000 000 kW. to divide the area into two equally loaded sections, interconnected only through the 100 kV. ring main.

There is much more in the paper which is well worth study. It shows that it is still worth while to plan for the more economical supply of large areas, rather than to make patchwork alterations to serve immediate needs.

CHAPTER VIII

RURAL DISTRIBUTION SYSTEMS

RURAL areas are "thin" from the supply undertakers' point of view. In general the designer is between two risks: first, that the revenue may be inadequate to the capital spent; secondly, that if the load grows beyond expectations, the initial work may be partly wasted. Whilst one must look ahead, it is safer at first to spend only the money essential to provide for the load to be expected within a limited period. Further expenditure will be easier to justify if the initial work has at least met the capital charges. It is rarely wise to lay out money which cannot earn a return in five years. Few investors care to bury capital for so long. Governments and other immortals can take longer views; and may be influenced by other than strictly economic motives.

The outstanding characteristics which affect the planning of a rural distributing system are, first, that the loads are scattered; secondly, that it is difficult to forecast the magnitude of the loads, and the rate at which they will accrue in future.

Preliminary canvassing may elicit some firm promises, but usually nothing very definite until supplies are available.

Where there is no gas supply, it may be expected that most people will be ready to adopt electric lighting if the price is at all reasonable, and the cost of wiring the smaller houses is met by some form of assistance. The lighting load can therefore be forecast with fair accuracy. Industries of the factory type already established have to be shown definite advantages in return for the cost of changing over from their own power plants to a mains supply. They may be reckoned on as eventual consumers, but at what future date is problematical. The same considerations apply to the farmers' requirements, usually actually met by petrol and oil-engine sets, including tractors. A more promising field is heating for dairy water, chicken incubators, etc., which are not so conveniently served by alternative agents.

The best one can do for forecasting loads of such kinds is to utilize the experience in rural areas where electric power is already in use; this will give some reasonable elements for

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estimating what will be required within, say, five years, and for plotting on a map the probable load distribution.

At present, overhead lines are the only economical means of supply. One writes "at present" because some trials are in progress of underground cables laid cheaply by a plough resembling somewhat the "mole-digger" used for laying drain tiles. The cables are three-phase armoured for 11 kV. Such cables and methods of laying may be cheaper than overhead lines, more reliable and less expensive in maintenance. It may be added that the plough works well on grass verges, unpaved margins and footpaths, as well as across fields, etc. Wayleaves may be more readily obtainable than for pole and mast lines across private property; the avoidance of the delays inseparable from the obtaining of consents for overhead lines—even along roads—is worth something. In plans for any new area it will be advisable to examine this development.

The sources of supply will usually be known, either generating stations or substations on a Grid or other e.h.t. line.

The pioneer lay-out should be designed to pick up all the load anticipated for some years of development. Most of the loads will be along main roads. Whether the lines should follow main roads entirely or cross private property and take by-roads in part, depends upon the topography of the area. Picking up all the loads on a minimum total length of line is to be aimed at. The minimum length can be traced on a map on which the load positions are plotted. The actual routes adopted will depend a good deal upon the facility of obtaining consents and wayleaves. Whilst the main roads may provide the shortest routes, the cost of meeting the views of the road authorities, the presence of Post Office lines along the roads, and objections from various quarters, may very well make routes across agricultural property preferable. Judging from what one sees in already electrified rural areas in this country, crossing lands instead of following the roads has recommended itself to many undertakers.

The approval of the Minister of Transport has to be obtained for all overhead lines; it is not given as a general approval over a defined area, but to specific routes shown on a map and accompanied by a specification of the construction proposed. Local and road authorities have to be notified in advance; they can submit objections, and the Ministry may hold a public inquiry if the objections warrant that course. Wayleaves

across private property are also the subjects of public inquiries if the property owners object to granting them. Hence the selection of routes requires careful study, inquiry, and some diplomacy, for minimizing opposition and delays.

As regards the type of network, two-way feeding of all sub-stations should be aimed at—ring or looped radial feeders for example. If the system is to be fed from two or more sources, the main lines may be laid out to interconnect such sources. Obviously the lay-out will be governed by the topography of the area. The pioneer lines should at least be traced to facilitate eventual development into ring or loop form, even if initial expenditure to that end is not justifiable. A meshed network of overhead h.t. lines is not to be recommended.

The most suitable voltage is probably 11 kV. That pressure can be used to supply small transformers of the pole type. Insulation and switchgear are less expensive and switchgear more readily accommodated than if 33 kV. were chosen. On the other hand, the capacity of the lines at 11 kV. is much greater than at 6 kV. or smaller pressures; and the cost of switchgear, insulation, etc., not much greater. Large villages and small towns offering a load of at least 500 kW. readily served from one substation, may justify a 33 kV. branch from a Grid line or substation within reasonable distance. Such a place will then be a convenient position for the supply to some of the 11 kV. lines.

The Electricity Commissioners' Regulations set out certain minimum requirements for overhead lines. The smallest copper conductor permitted is No. 8 S.W.G. having a cross-sectional area of 0·0201 in.^{2}, a breaking strength of not less than 1 237 lb., and a weight of 409 lb. per mile. The supports must have a factor of safety of 3·5 when subjected to the stresses of conductors ice-loaded to a radial thickness of $\frac{3}{8}$ in. and a wind pressure due to a 50 m.p.h. gale at a temperature of 22° F. The minimum height of conductors from the ground is to be not less than 19 ft. across a road; wooden poles must be of creosoted red fir. For low-tension lines in general, and (by special permission only) some high tension lines, the radial thickness of ice loading specified may be reduced to $\frac{3}{16}$ in. Practically all rural lines will fall under the high-tension rules.

These requirements, taken in conjunction with the lengths of spans, have the effect of imposing some minimum cost for a line along a given route. Longer spans involve fewer but

taller and stronger poles, also fewer insulators, arms, etc. It is possible, having sufficient price data, to arrive at the most economical length of span for any given route. Practically, pole positions and span lengths are not usually matters of free choice; the curves and angles of roads, positions of buildings, and positions of hedges across country, determine them to a great extent. At the same time, it is worth while to work out the most economical span, and keep to it as far as possible.

Other materals than copper are rarely used for lines of the kind under discussion. Steel conductors are lighter for the specified breaking strength—they allow of longer spans and lighter poles—but the much higher resistance of steel confines its use to short, lightly loaded lines, such as services to isolated consumers.

Plotting the line routes and the positions and magnitudes of the loads to be expected, in say, five years' time, will show how much of the system can be of the minimum structure type, i.e. with conductors of No. 8 S.W.G. copper, without exceeding such limits of current density as are imposed by either voltage drop or economic considerations. Heating will not be a limiting factor for such a line. The voltage drop to the most remote load, or annual economy on Kelvin principles, may be decisive; these points and other details affecting economical design will have to be considered specifically in each case.

There will usually be possible consumers at considerable distances from the main lines. Branches serving them are essentially services. Strictly, consumers should defray the cost of services which serve themselves alone, either by lump sum or rental payments, a requirement not always agreeable to the consumer; but it should be possible to get some contribution. One way sometimes adopted is to get the consumer to supply and erect the poles at his own cost. Services may be of a minimum size of No. 10 S.W.G. copper, by the regulations.

Generally the l.t. distributors will be confined to short runs from each transformer to a few consumers, frequently only one. In villages small l.t. networks may be necessary. L.t. mains may be run on the same poles as the h.t. with certain precautions, so far as these serve. Quite a good job of the l.t. distributors may often be made by running them on brackets on the house walls—under the eaves, for example; methods more common in continental countries than here.

If there are outlying houses, farms, etc., to be served, the alternative of l.t. branches from the village, or h.t. branches with transformers at the loads, should be considered. Voltage regulation will be simplified by keeping the radius of l.t. distribution short and fairly uniform.

The subject of the design of distributing systems for rural areas in this country was dealt with very ably by Messrs. E. W. Dickinson and H. W. Grimmitt in a paper published in the *Journal of the Institution of Electrical Engineers.** The paper and discussion which followed give views from different angles and some examples from practice. Reference should also be made to descriptions of the rural area systems served from Norwich (Messrs. V. A. Pask and R. W. Steel read a paper on the Norwich area with results of its working before the Annual Convention of the Incorporated Municipal Electrical Engineers Association at Brighton early in June, 1937, of much interest); and from Bedford, which have been published in the technical press.

In this country one has to study economic design within the framework imposed by the current Regulations issued by the Electricity Commissioners and the Postmaster-General, which settle many elements of the cost of overhead line construction. A great deal of information of what can be done and is done under different, sometimes freer, conditions has been published in the electrical papers of the United States, continental countries, and of this country with reference to work in the British Dominions and Colonies. There are always some considerations of mechanical stability and safety which constitute a framework of minimum structure, etc., for the economic design, even if there are none imposed by outside authority.

* Vol. 70 (1931), p. 189.

CHAPTER IX

THE BRITISH STANDARD SYSTEM: REGULATIONS AFFECTING DISTRIBUTION LAY-OUTS

THE British Standard System of distribution is a three-phase four-wire system at a frequency of 50 cycles per sec., 400 volts between phases, and 230 volts between each phase and the neutral. The neutral conductor is earthed at some definite point of the network.

Consumers' services are either single-phase at 230 volts, or three-phase according to the magnitude and nature of their loads. Services to industrial works, etc., with motors of more than a few horse-power are usually three-phase. Large consumers for lighting, heating, lifts, etc., should have three-phase (i.e. four-wire) services if their loads can be conveniently split up for balancing among the phases. There are some special Regulations applying to services which have more than 250 volts between any two conductors, e.g. three-phase at 400 volts; these refer in the main to the wiring and apparatus on consumers' premises.

The voltage range permitted by the Commissioners' Regulations is now 6 per cent on either side of the declared voltage. At 230 volts the total range 6 per cent above to 6 per cent below is therefore from 244 volts to 216 volts, neglecting fractions. If the whole variation were to occur in the distribution mains, there would be a permitted difference of 27 volts between the nearest and the farthest services at peak load times. The phase-neutral voltages at the transformer terminals would have to be, say, 245 volts at such times.

This may be partially effected by an increase of pressure at the generating or grid substation, but it is not practicable to regulate there in such a way that every distribution transformer receives a primary voltage corresponding to its individual load. Voltage regulation by tappings or induction regulators at the distribution substations is practicable; there are now available automatic regulators for unattended substations. All such apparatus adds to the first cost and to the cost of the losses as well as to their magnitude. From the strictly economic point of view it will generally be advisable to calculate

for a smaller range than the 12 per cent now permitted. When quality of service is considered, the reasons for a smaller range are accentuated. The effects of voltage variations on consumers' appliances have been investigated by Messrs. E. B. Wedmore and W. S. Flight with results set out in Report Z/T 41 of the British Electrical Research Association. Some of the results are—

Lamps. Five per cent drop results in a reduction of about 20 per cent in the light and an increase of about 60 per cent in the life; 5 per cent over-voltage results in an increase of about 20 per cent in the light and a reduction of about 50 per cent in the life. Hence 5 per cent each way from the normal voltage may well give rise to complaints of poor lighting on the one hand, and of excessive cost of lamp renewals on the other.

Cooking Appliances. The effects are also serious, 5 per cent low means a large increase of boiling time of kettles, heating up of ovens, and some waste of energy. Radiant heaters which normally work at a bright red heat have their lives approximately halved by an over-voltage of 6 per cent. Complaints of such effects are not uncommon. Cooks are particularly intolerant of anything which upsets their accustomed timing of operations.

The Report mentioned has some records of voltage variations observed on a number of systems, analyses of the causes and suggestions for better practice.

On all counts it will be better to plan for not more than 4 per cent up and down from the declared or normal voltage at consumers' premises.

Assuming that some standard sizes of cables have been decided upon, the voltage to the end of a distributor can be readily calculated for any known or assumed distribution of the load along it. The drop is the sum of the product of the current in each length and the resistance of the conductor of that length. Hence one can express the loading which will give a certain drop as the sum of a number of products of amperes by yards, a constant for that size of conductor. For example, a four-wire distributor of 0·25 in.^{2} conductors has a resistance per conductor of about 0·103 ohm per 1 000 yd. If the permissible drop is 8 per cent of 230 volts, i.e. 4 per cent up and down, or 18·4 volts the limiting product of current by yards is 178 000. If the loads and their positions are known,

the sum of the products for the several lengths between loads and the currents carried over each length must not exceed that total. This assumes that the load is perfectly balanced between the three phases, so that there is no current in the neutral conductor; a condition not to be relied upon. If there is a known or calculable unbalance, the resultant ampere-yards product in the neutral must be added to the products sum in the most heavily loaded phase conductor. Another point to be noticed is the current in the length between the nearest load and the substation. That must not exceed the safe currents on heating considerations. For a 0·25 in.² armoured cable laid in the ground, the limiting current is of the order of 325 amperes per phase for a working temperature of 65° C. Hence such a cable would be thermally overloaded with a current of 356 amperes carried 500 yd., although the amperes × yards product would still be the permitted 178 000. In fact the higher temperatures at the maximum safe loading increases the resistance of the conductors by about 20 per cent making the limiting amperes × yards about 150 000 instead of 178 000. Although the condition of maximum safe loading over the whole length of a distributor is not likely to occur, it will be advisable to figure on that basis.

It is rarely possible to foretell the conditions as regards phase balancing. The worst condition that can arise is all loads on one phase which just doubles the drop and halves the amperes × yards constant.

In the case of a distributor fed from both ends, the length to take is approximately to the mid-point between the two substations. The point of lowest voltage on such a distributor can be found by calculating for feeding it first from one end and then from the other. The point where the drops equalize under the two conditions will be the point sought, and the drop there will be that resulting when the same loads are fed from both ends.

The Kelvin relation for a distributor supplying a number of loads can only be strictly complied with by stepping the conductor section in accordance with the load steps. That is not usually practicable for distributors; it may be so for long dead-ended lines such as h.t. feeders supplying a number of substations where the loads may be taken as the capacities of the substations. In any case the annual costs resulting from a practicable lay-out in respect to limited voltage drop and other

factors can be compared with those resulting from compliance with the Kelvin relation, supposing that to be possible. Such comparisons are rather out of the scope of a chapter dealing with regulations ; it must suffice here to point out the connexion between regulation and distribution economics, and that economic current density may have to take a second or third place after voltage drop and heating limits.

As regards the heating limits, it should be noticed that if a number of cables are bunched, as may be unavoidable at the exits from a substation, the safe current capacity is diminished. The British Electrical Research Association has published tables which give the limit currents under various proximity conditions.

When a distributing system is planned with certain sizes of cables, the number of such cables leaving each station gives the transformer capacity which can be usefully installed there, viz. that corresponding to the total heating current limit of all the cables. The permissible voltage drop puts a limit to the product of current and distance for each cable. If all the cables were equally loaded uniformly along their length, the substation would have a definite radius of supply shorter or longer as the density of load over the area were greater or smaller. Hence it may be said that the size of cables chosen, the peak load density over the area, and the permitted pressure drop, govern the spacing and capacities of the substations.

The Commissioners' Regulations relating to the laying of mains, the equipment of substations, safety precautions, etc., are not factors in the economic design of underground systems ; they rather prescribe conditions with which the design must comply.

Authorized undertakers have a general right to lay mains, etc., in the public streets and roads of their areas of supply. Plans of such works have to be submitted to the road authorities in advance, and those authorities may require alterations in the plans. If the alterations will add to the cost, or are objectionable on technical grounds, the question may be referred to the Commissioners. The road authorities may absolutely veto structures on the streets above their surface, e.g. switch pillars, transformer kiosks, and the like ; consents for such structures are rarely given to Company undertakers, though Municipal electrical departments seem to use them pretty freely. Undertakers have general powers to lay mains in

streets, etc., in private ownership, subject to agreements with the owners in certain respects. Frequently special provisions are put into Orders with respect to certain named roads, bridges, and so on, not repairable by road authorities. The differences which arise between undertakers and road authorities are most often on the position of works in the streets which affect the cost of making good. The undertakers' engineer will select, as far as is compatible with the object of the works, positions which involve the least expenditure, e.g. by arranging that the paving to be broken through is of the less expensive kind, rather than the more expensive ; the road authority's engineer may, on the other hand, have reason to prefer the more expensive route or line.

It is possible to obtain compulsory wayleaves for underground mains across private property of certain kinds, after a somewhat protracted procedure of inquiry, etc. There is rarely much occasion to require such passage in urban areas. Of course, consent may be obtained by agreement : it is also possible to acquire property for the construction of substations compulsorily by "Special Order" also a somewhat protracted precedure, not to be entered upon lightly.

CHAPTER X

DETAIL DESIGN BASED UPON LOAD FORECASTS

PREVIOUS chapters have dealt broadly with the lay-out of an area as a whole, and the general lines of planning for economical operation having reasonable regard to the future.

In this chapter an attempt is made to show how the same principles can be applied to portions of an area, say a street of a particular class of premises. This kind of problem arises repeatedly in established undertakings when extensions have to be made. Then the question is, what load can be reckoned on, what can be properly spent, and what return can be anticipated?

If one could know in advance the amount, distribution, and characteristics of the load, the problem would be simplified. Such information is not generally to be had. The best one can do is to utilize such data as are available from experience in similar areas. Complete data from existing undertakings are scanty. It has not been the usual practice to measure and record load characteristics in the detailed way which is desirable. The most complete survey of the kind known to the author is embodied in the paper read by Messrs. Woodward and Carne before the Institution of Electrical Engineers in 1932.*

In the discussion on the paper several speakers added data for particular areas, among which a contribution by Mr. J. N. Waite (of Hull) should be specially noticed. In these cases the costs of the supplies are worked out from generation to delivery and allocated, taking into account the internal diversity factors and the system diversity factors for each class of consumer and load. It is much to be desired that data of these kinds should be more generally collected and published.

Figures of *per capita* consumption over large areas, whilst of value for certain purposes, do not give much information for forecasting consumption in detail. Some day it may be possible to follow the water engineers who can foretell with considerable accuracy how many gallons per head per day will be needed for districts of defined types, but that day is not yet.

* *J.I.E.E.*, Vol. 71 (1932), p. 852; and Vol. 72 (1933), p. 358.

The best one can do is to use the data available for plotting out the number of positions of all the premises which will become potential consumers, and allot to each a load of such magnitude and characteristics as may be warranted by experience. That will result in a plan or map of possible loads. The individual maximum demands, and the diversity factor between the individual consumers, are essential data for the accurate solution of the problem.

A slightly different problem than that of an extension arises when the growth of load in an area requires an increase of distribution capacity. Then it is a question of the most economical way among possible alternatives. The resulting lay-out is likely to be very different from that which would have been designed for the increased capacity at the outset, and probably less economical. That is a cogent reason for basing pioneer designs on the loads to be reasonably expected in some not too remote future, preferably by providing facilities for reinforcement which involve no great cost until they have to be utilized.

Records of consumption in similar districts are more or less useful as they are more or less complete. The minimum of information readily available is the total annual consumption of consumers of a particular class, or of the consumers in a particular area. That consumption divided by the number of consumers gives the average consumer of the class or in the area taken. As the individual consumptions, i.e. the meter readings of all the consumers, are presumably available, they allow of an analysis which is decidedly useful.

A "cumulative frequency curve" may be plotted between "number of consumers" and "annual consumption." Such a curve is shown in Fig. 4. The axis of ordinates is scaled in number of consumers, the axis of abscissae in the number of kilowatt-hours taken annually by individual consumers, so that each point on the curve represents the number of consumers who take not less than the number of kilowatt-hours represented by the abscissa of the point. The area of any horizontal strip represents the number of kilowatt-hours taken by the number of consumers represented by the vertical height of the strip. The total area of the curve represents the total number of kilowatt-hours taken by all the consumers.

For simplicity of illustration it has been assumed that a total of 100 consumers can be divided into 14 groups, each group

of 5 or 10 consumers taking an equal number of kilowatt-hours as shown by Table III below—

TABLE III

No. of Consumers in Group	Consumption of Each individual Consumer	Consumption of Group	Cumulative Consumption	Percentage of Total
5	3 000	15 000	15 000	14·74
5	2 500	12 500	27 500	27·03
5	2 000	10 000	37 500	36·85
5	1 500	7 500	45 000	44·22
5	1 200	6 000	51 000	50·12
5	1 000	5 000	56 000	55·03
5	900	4 500	60 500	59·46
5	850	4 250	64 750	63·63
10	800	8 000	72 750	71 50
10	750	7 500	80 250	78·87
10	700	7 000	87 250	85·76
10	650	6 500	93 750	92·12
10	500	5 000	98 750	96·29
10	300	3 000	101 750	100·00

This curve is a stepped one. In practice there would be more steps, probably there would be few instances of two or more taking the same quantity per annum. The curve would not end so abruptly at 300 and 3 000 kWh. if it covered a few thousand consumers. It would become fairly smooth, as indicated by the dotted line joining the points of the steps. Since the area of the curve represents the total consumption, a horizontal line dividing it into two equal areas cuts the curve at the abscissa representing the average annual consumption per consumer, which is 1017·5 kWh. in the example. The corresponding ordinate, approximately 25, represents the number of consumers who collectively take half the total, individually not less than the average; the remaining 75 take the other half with less than the average.

The curve can yield more information than that. The intersection by a horizontal line at half the height gives what is known as the *median*. The abscissa of the intersection is the consumption of the middle consumer, and there are equal numbers of consumers who take less and more than that particular one. Another average of more interest is what is known as the *mode*. That is the annual consumption used by the maximum number of consumers, and is indicated by the
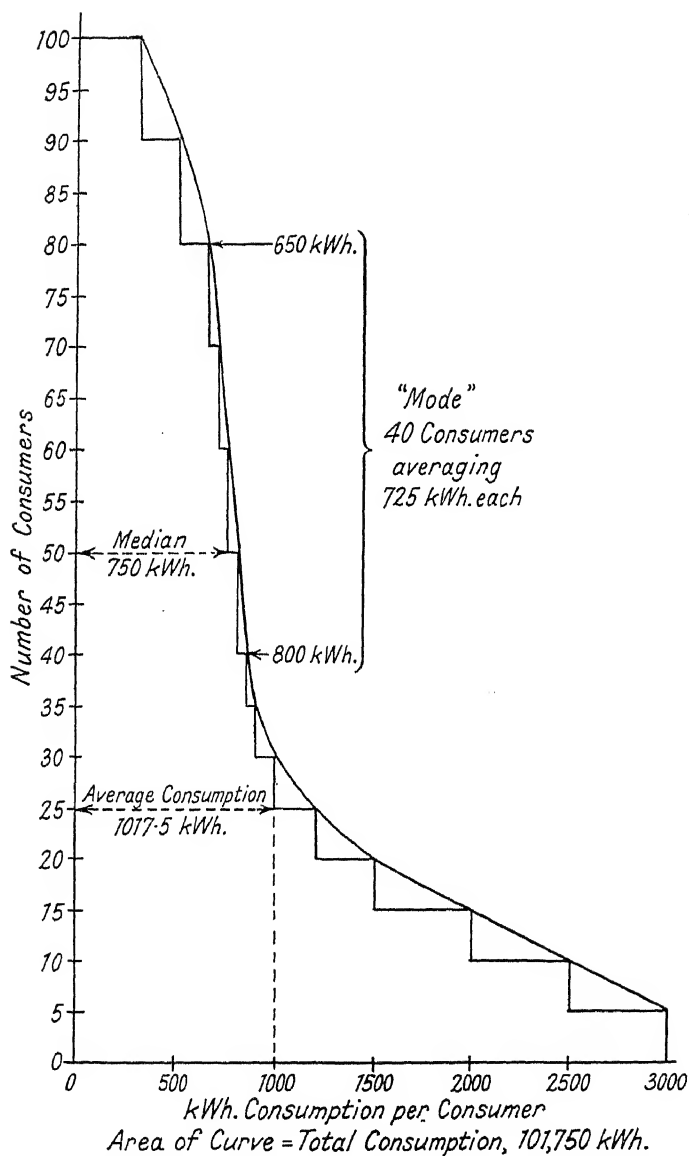
FIG. 4. CUMULATIVE FREQUENCY CURVE DERIVED FROM TABLE III

position of the maximum slope of the curve. It may be regarded as the consumption of the consumer who is neither very economical nor very extravagant. It represents what the average consumer of the class is willing to pay for electrical energy. Such a curve is most useful for forecasting if made for a definite class of consumer supplied at a definite tariff. In the curve of Fig. 4 the maximum slope is constant between 900 and 650 kWh., covering 45 consumers. The mid-point is 775 kWh., so that in this case it may be reckoned that some forty consumers will take that average. That is useful for forecasting as an amount which may be expected for each consumer, supposing that those of the data obtained and those to be supplied are fairly comparable.

A curve derived from the cumulative frequency, known as the *Lorenz* curve is a plot of the percentage of the total consumption against percentage of total consumers. Such a curve is shown in Fig. 5. It classifies consumers by their percentages of the whole consumption; thus the 10 per cent of largest consumers take 27 per cent of the total, and the 10 per cent of smallest consumers only 3·7 per cent.

These curves have further uses for the calculation of the probable results of tariff changes, two-part tariffs, etc. Comparative curves over the same group of consumers or the same areas in successive years show the results of tariff changes, selling campaigns, etc.

The author desires to acknowledge that so far as he knows, the application of these statistical methods to consumption analysis is due to Mr. C. R. Cooper and Mr. G. W. Stubbings.

If, in addition to meter readings, the maximum loads and diversity factors associated with each class of consumers are available, forecasts of the requirements for similar districts or classes can be made with much more security of approximation to the elements needed for the most economical design.

Messrs. Woodward and Carne's paper above referred to gives some very valuable information of the kind.

Five years is about the minimum period one should consider for estimating probable loading. Telephone engineers have developed the application of the theory of probabilities to their own needs in great elaboration, and there is an extensive literature on the subject. They aim at providing cabling for fifteen years' progress: they have good reasons for taking so long a view because they have to provide a separate pair for
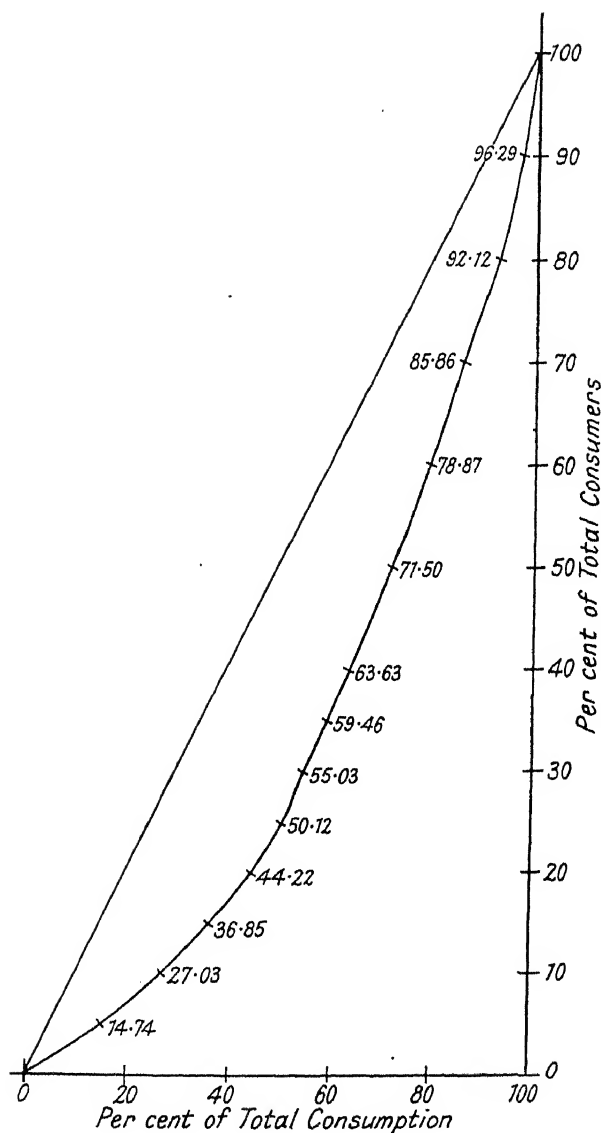
FIG. 5. LORENZ CURVE DERIVED FROM TABLE III
The chord to the curve would be given by equal consumption by all consumers.

every subscriber along the length of a cable, hence an under-estimate would entail the laying of an additional cable with all the repeated expense of the street works before that first laid had earned replacement cost. It is *as if* the distribution of electrical energy required separate services for each consumer right back to the distributing centre. The methods of forecasting future demands developed by the telephone engineers are worthy of consideration.

To-day one may reckon that all the houses above a certain rental in a street in which mains are laid will require services for lighting, at least, within five years of the supply being available. It is less certain how many of them will have cooking, water-heating, and electric fires. Much depends upon the tariff and the facilities offered for the purchase and hire of appliances. Data from similar districts will give some hints. The use of attractive tariffs and salesmanship methods may be expected to give, in the course of five years, some average load for premises of each kind as a basis for distributing estimates.

Assuming that forecasts have been made from the best data available, a single example of their application may be considered. It is a deliberately simple example.

The unit taken is half a mile of road in a residential suburb, lined by houses rented at about £60 per annum. Each house will occupy a road frontage of some 30–35 ft. Allowing for street crossings, etc., there will be some 80 houses on each side, or 160 houses altogether. Each house may be expected to have a connected load of 800 watts lighting; 3 000 watts of cookers and the like, and 1 000 watts of water-heating load. The peak demands of each house will probably be—

Lighting	.	.	.	.	400 watts
Cooker	.	.	.	.	3 000 ,,
Water heating	.	.	.	.	1 000 ,,

These peak demands will not be simultaneous; their time incidence will vary considerably. Observation shows that the diversity factor of cooking loads is of the order of 8, so that the group maximum may be taken as $3 \times 160/8$ or 60 kW. The diversity of storage water heaters is probably about 5, making the group peak about 32 kW. The lighting peak may be taken as coincident in all the houses, making the peak value 0.4×160 or 64 kW. Although the peaks of these three kinds of load are not found to coincide in time, the diversity

factors taken for the cooking and water heating may be considered to allow for that; so that the peak load of the 160 houses may be reckoned as 156 kW., which is 156/768 = 20·3 per cent of the connected load. This is about the ratio of peak-connected load found to occur in British undertakings, so that whilst the constituents may be open to question the result coincides with general experience. In round figures the maximum demand for the half-mile of street may be taken as 150 kW. With a three-phase four-wire distributor, the loads balanced between the phases, and the distributor fed at one end, the current per phase will be 217 amperes. If a distributor is laid along only one side of the road, it is certain that by the time all the houses are connected there will have been more excavation and making good in road crossings, and spreading out from crossings on the other side, than if a distributor is laid on each side; as well as very considerable lengths of branch cables. Hence a four-conductor cable should be laid on each side, each reckoned for 109 amperes per phase at the feeding point. It would be advisable to connect across the far ends of the two cables by links or fuses.

Two standard cable sizes of the suitable class have conductors of 0·1 in.² and 0·06 in.² respectively. As regards heating, the 0·06 in.² cable has a maximum safe capacity of 169 amperes so that it could carry the feeding end load of 109 amperes comfortably. The 0·10 in.² has a maximum safe capacity of 229 amperes per phase, so that it would be lightly loaded with 109 amperes. As regards voltage drop, the resistance of the 0·06 in.² conductors is 0·36 ohm for the half-mile. Assuming symmetrical distribution of the loads along the length of the cable, the voltage drop will be the same as for half the total current over the whole length; say 55 amperes × 0·36 ohms = 19·8 volts, which is 8·61 per cent of 230 volts. Hence if the substation voltage is regulated to 4 per cent above normal at full load, the volts at the far end will be a little more than 4 per cent below normal, and those at the feeding end 4 per cent above. Thus the 0·06 in.² cable would meet requirements under the new 6 per cent permissible range. But the conditions assumed are the most favourable possible, viz. that at every tapping the phase loads are balanced so that there is nowhere any current in the neutral conductor, and that the loads are all of unity power factor—conditions not likely to prevail. Hence it may be said that at the half-mile distribution radius the

load of 109 amperes per phase at the feeding end is just about the practicable limit for a 0·06 in.² cable. If the loads were heavier towards the far end, the voltage drop would be too great. For the case considered it might be put down without qualms if feeding from the other end would be easily practicable in case of the load growing beyond that expected, but considering all the probabilities it might be wiser to lay a 0·10 in.² cable or to arrange to feed from both ends at once. The estimated load—150 kW.—is just twice that permitted by the regulations for an under-street substation; so that single-end feeding means a substation of a more expensive kind. It may be cheaper to put down a 75 kW. under-street substation at each end than a 150 kW. above ground substation at one end. Evidently the additional cost of the h.t. feeder to the second substation has to be reckoned with. Supposing, however, that the economic choice lies between 0·06 in.² cable fed at both ends and 0·10 in.² cable fed at one end only, the difference between the cost of one mile of those cables is of the order of £200 at to-day's rates; the difference between the cost of two substations of 75 kW. each and one of 150 kW. should be less than that. The h.t. feeder cost may or may not therefore decide the question. If the two-end feed is too expensive, the 0·10 in.² cable will be the wiser choice. That would certainly be the case if cross-streets out of the main road were likely to be built up and to require supplies within four or five years.

It remains to be considered what will be the probable annual costs of the losses and charges upon the cable. Taking the cost of a mile of 0·10 in.² cable at £600, and the annual charges at 10 per cent or £60 per annum, and the cost of the energy put into it as 0·6d. per kWh., the Kelvin relation puts the cost of the losses for 100 per cent load factor load at £20 per annum per phase conductor. At 0·6d. per kWh., that is 8 000 kWh. per annum, or 8 000/8 760 = 910 watts; taking the resistance of the mile as 0·45 ohms the current is very closely 45 amperes.

$$(\sqrt{910/0·45}) = \sqrt{(2\ 020)} = 45 \text{ very nearly})$$

That is for 100 per cent load factor and for the current flowing the whole length. A residential suburban load is likely to have a load factor of 25 per cent. The highest (r.m.s.) form factor for 25 per cent load factor is 2, i.e. the current density would be doubled and the loss rate quadrupled for one-quarter of the time. That would make the current 90 amperes and the loss

rate 3 645 watts per mile with a voltage drop of about 40·5 volts per mile, or just over 20 volts for the half-mile. Those values are, however, for the whole current flowing the whole length, whereas in the distributor the load tapers off from 109 amperes to nothing. The peak current at the feeding end— 109 amperes—would be above the economic density, but the average along the whole length would be below the economic density; with a voltage drop at the end of only about 12 volts, quite moderate. The loss rate would be approximately the same as that for a whole length current of 78 amperes or 2 737 watts per phase, totalling to 5 996 kWh. worth £14·99 per annum against the Kelvin rate of £20 per annum. In practice the form factor would be less than 2 and the annual losses also less than those calculated above. Certain assumptions made, e.g. that the current tapers off uniformly (whereas it goes by steps), and that the load is perfectly balanced between phases at all times, can be put against the too high form factor, and the calculated figures taken as not far from the truth.

The consumption due to a peak load of 150 kW. and a load factor of 25 per cent is 328 500 kWh. per annum; the calculated annual losses are 5 996 × 3 = 17 988 kWh. costing £45, so that the physical efficiency of the cable would be 328 500/346 488 = 94·8 per cent, and the cost £105/328 500 = 0·0765d. per kWh. delivered to the consumers.

As the capital charges item is greater than the losses item it follows that a smaller cable would give a lower total annual cost, if its cost were proportionate to its copper section.

The 0·06 in.^{2} cable costs approximately £400 per mile; its losses on the same loading would be in the ratio of the respective resistances, viz. 0·71/0·45 = 1·57, and their cost £45 × 1·57 = £70·65, making the total of charges and costs £110·65 per annum, a little more than with the 0·10 in.^{2} cable. The difference is not great; but considering also the greater capacity and the better regulation, the larger cable is well worth its cost from all aspects. If the cost of the energy delivered to the cable were lower, the smaller cable might be the more economical for the case considered.

There would be some saving in the capital charges by putting down the larger cable for half the length and the smaller cable for the other half; but unless the road were certain to remain a dead end as regards electrical requirements for a long time, that would hardly be wise.

So far only the cable costs have been discussed. The costs of laying are often greater than that of the cable. They are not within the complete control of the designer. The major cost is usually that of making good the paving surface, which varies within very wide limits. But the trenching cost can also vary widely, between digging in soft soil and cutting out concrete. There is usually no choice as regards distributing cables; they have to go along the footpaths. There will rarely be good reason for laying distributors in ducts of any kind. Exceptions are at street crossings, near substations whence many cables have to be got away, or where it is clearly advantageous to be able to put in more or larger cables without breaking up expensive paving or interfering with heavy traffic. For the crossing of streets with heavy traffic, screwed wrought iron tubing is a cheap and strong form of ducting; it permits of curving around obstacles easily.

For the assumed half-mile of suburban road there will be a mile of trenching, back-filling, and making good of surface. If the footpaths are tar-macadam, or stone flagged, the cost of this work may be about 4s. per yard run; it may be more or less in any given case, but not much less anywhere. One mile at 4s. per yard run costs £352; jointing, service boxes, supervision, etc., will cost about £250; then the total cost of the mile of distributor laid jointed, with service boxes, etc., will be—

Cable . .	£600
Trenching, etc.	352
Jointing, etc. .	250
Total	**£1 202**

The cable costs barely half the total. The annual charges on it have been taken at the rate of 10 per cent: since replacement at the end of its useful life will involve excavation, making good, etc., as for the original laying, the same rate is appropriate for those costs. There will also be some expense for maintenance. Then the total costs and charges pertaining to the half-mile distribution will be—

Cable, losses and charges	£105
Laying, etc., charges	60
Maintenance, estimated	10
Total . .	**£175**

Divided by the total kilowatt-hour delivered per annum this comes to 0·127d. per kWh.

It is of interest to compare these figures with some given in Messrs. Woodward and Carne's paper above referred to for the capital cost of l.t. distribution to domestic consumers; Class C in their schedules.

	Estimate	Woodward and Carne
Cost of mains, laid, per kW. demand .	£8·017	£9·950
Cost of mains, laid, per consumer .	7·50	6·950

to which may be added

Annual cost per kW. demand . .	£1·093	—
Annual cost per consumer . . .	1·086	—

Messrs. Woodward and Carne do not give comparable figures for the second pair of costs. It seems that the estimates in this chapter are of the right order of magnitude; they are only put forward as examples of a method which might have been represented in a more generalized form by algebraical formulae.

Assuming that the base cost rate of 0·6d. per kWh. covers all generating and transmission costs with a margin of profit up to the delivery point of the substation, the financial results of different average prices for the sales are readily calculated, e.g.—

Sales (kWh.)	Average Price (d.)	Gross Receipts (£)	Less l.t. Costs (£)	Net, less 0·6d. per kWh. plus l.t. Costs (£)
328,500	0·75	1026	851	30
,,	0·80	1095	920	99
,,	1·00	1369	1194	373

The last column gives the annual amounts accruing to the profit revenue of the undertaking as net return for the expenditure of £1 202 on the mains extension; the capital charges in the £175 of distribution costs being regarded as necessary to cover interest, depreciation, etc. At an average price of 0·75d., the return of £30 per annum, may be regarded as less than a reasonable margin of profit. The £373 resulting from an

average price of 1d. per kWh. looks excessive. There will be some capital and working expenses not included in these estimates : services, meters, meter reading and book-keeping, which should be met from meter rents or fixed charge elements in the tariff.

Estimates of this kind of the net revenue from a certain length of main are subject to certain considerations. In the first place the full amount will not accrue immediately. If the location is within the "compulsory area" of the undertaking, there is no alternative to giving a supply ; even if outside that area there may be conditional obligations which become operative. The tariff to be applied has a great influence upon the consumption. No consumption rate of over 0·75d. is likely to attract cooking, much less water-heating, loads of appreciable amount. Experience shows that a two-part tariff with a consumption rate of 0·75d. or 0·5d. attracts a good deal; the fixed charge then becomes a base line revenue which should logically cover the standing charges of the undertaking properly allocatable to each consumer. In the case taken, the part due to the distributor has been shown to be the return on £7·5, but that is only a fraction of the total cost of the plant between the generating station or grid delivery point and the substation delivery, which has been assumed to be included in the cost of 0·6d. per kWh.

The point to be emphasized is the relation between the consumption of a particular class of consumer and the tariff. For the class considered, a flat rate of, say, 3d. per kWh. will only obtain a lighting load; the load factor of such a load will be more like 10 per cent than 25 per cent, which alters most of the values used for the calculation. Estimates of consumption must be based upon the tariff contemplated to be of any value. Here the discussion borders on ground more fully dealt with in a subsequent chapter.

It may be noticed that if the load factor proves to be higher than that reckoned on, the costs will be favourably affected. For instance, if the load factor of the half-mile proved to be 30 per cent without increase of maximum demand, the losses in the cable could not be increased by more than 20 per cent, i.e. they would remain at the same value per kilowatt-hour, while the capital charges per kilowatt-hour would be reduced by 15 per cent approximately.

The case taken was deliberately simple, and worked out from

first principles. In an existing undertaking it may be supposed that such general data as economic current density—or rather the economic current and the amperes-×-yards product for the permissible voltage drop for each size of cable, with the consequent economical radius of supply from substations—would be known. It would be useful to work out a few examples of typical streets and their loadings for reference.

It will be observed that the 0·1 in.² cable would supply economically a more densely loaded street if it were fed at more frequent intervals. For loading of the order discussed, voltage drop is a more effective determinant of cable size than either the heating limit or the cost of losses. Voltage drop and the cost of losses operate in opposed directions. Low load factor, i.e. poor time utilization of the cable, calls for keeping down the size, entailing a voltage drop at full load which may be excessive. Heating is not a serious limitation in isolated mains of the capacity discussed. It is otherwise where many mains are in close proximity, in larger cables, and in extra-high-tension mains.

Calculations of the nature outlined based upon the best estimates available of the load density per unit length of street in different districts will probably show that the requirements of most of them can be met by adopting a few of the standard sizes of four-core cable, say, between 0·1 in.² and 0·25 in.²

On the same lines as set out for the 0·1 in.² cable, one of 0·25 in.² can carry 98·6 amperes per conductor as its most economical current for 100 per cent load factor when the drop will be about 17·75 volts per mile. For the most severe, 25 per cent, load factor load, the economic current will be 197 amperes per conductor, when the voltage drop will be 35·5 volts per mile.

It will be noticed by those familiar with current practice that the economic current densities calculated above are much lower than those commonly found in use. It seems that mains engineers have been inclined to economize overmuch in copper, with the result that mains losses are higher than they should be. This tendency dates from the time when lighting load predominated, giving a load factor of the order of 10 per cent. The extreme r.m.s. value of a 10 per cent load factor is 3·16, which would make the economic loading of a 0·25 in.² conductor about 311 amperes, very close to the thermal limit for a four-conductor cable, and giving a voltage drop of 54 volts per mile,

really more because of the temperature rise. At such a poor form factor the costs of the losses will be higher than has been taken, because of the effect upon the generating or bulk supply costs of the relatively high peak. One may also notice that copper has of late years been much cheaper than formerly, but subject to rather violent fluctuations during 1937 for example.

The most "peaky" loads likely to occur to-day are in blocks of commercial offices where the lighting load lasts for perhaps three hours in the winter and vanishes almost entirely for three or four summer months. Shopping districts where brilliant window lighting prevails from dusk until about 7 to 8 p.m. also have a poor annual load factor with high peaks about mid-winter. These unfavourable load features are in process of some improvement by the adoption of electric heating in offices and window and display lighting after closing time in shops. Office heating is likely to increase the peak load, but tends to improve the overall load factor. Lifts, air-conditioning, cooking in canteens and restaurants, fans, etc., also help. It is therefore good practice to encourage such loads by tariff arrangements. In districts of these kinds it may pay better to put down transformers at rather short intervals, and to switch some of them out after closing time and during the summer months, rather than to put down heavy distributing cables.

The lighting load of factories and workshops is also of poor annual load and form factors. The lighting comes on to the winter peaks. If such places also take power, the overall load factor may not be too bad, but the peaks have to be provided for.

The results of detailed local estimates of the kind outlined will be a plan of mains of definite sizes, with appropriate places for substations of definite capacities indicated by the convergence of the mains and their estimated loading. They are applicable to all areas of supply, whether extensions of old undertakings or entirely new ones.

Something should be said about the use of voltage regulators on distributing mains. In some cases it may cost less overall to install voltage regulators than to use a cable of sufficient capacity (or more substations) to keep the voltage drop within bounds. Such cases arise where distributors have become overloaded *quâ* voltage drop but not *quâ* heating, and where the loads are at long intervals but individually small, so that

the length of distributor per consumer is relatively great. The first case arises in existing networks on which the load has grown beyond the original estimates; the second case arises in outlying suburbs with houses at long intervals, approaching the conditions of rural districts.

The densely and overloaded network of the first kind of case would probably be best dealt with by putting down additional substations or more copper; but sometimes that cannot be done as soon as desirable.

The second kind of case, the long distributor not overloaded in the economic sense, but having an inadmissibly high voltage drop at peak load, can be dealt with by one of the booster regulators, perhaps an end booster. But in planning for such a distributor it would probably be wise to consider putting down a h.t. feeder with transformers under the street or in kiosks, and only very short l.t. cables. In the U.S.A. it is now common to run h.t. lines at from 2·3 to 11 kV. along such outlying suburban roads, and put in transformers for each consumer. Whether that can be done in this country depends upon local conditions not amenable to generalizations.

The most difficult areas for load forecasting are rural districts. In this country there is as yet but little available experience, but it is being added to rapidly. As a general rule one would run overhead 11 kV. or 33 kV. lines along roads and cross country to pick up villages, factories, and so on, of the minimum conductor sizes required by the regulations. Such conductors can supply substantial loads over considerable distances. There is not so much cost involved in augmenting the capacity of overhead lines as in augmenting the capacity of underground cables, so that under-estimates of the load involve less risk of unnecessary eventual cost.

CHAPTER XI

REINFORCEMENTS AND EXTENSIONS OF EXISTING SYSTEMS

WHEN the load on a portion of an existing system approaches the limits of either the permissible voltage drop or the heating of the mains at peak loads, the question of the most economical way of reinforcement needs consideration.

Where the distribution is by d.c. three-wire mains fed from rotary converting substations, the usual practice has been to lay new feeders from the substations to suitable points on the distributors. That is a costly plan, for it means adding conductors of the most expensive class in proportion to their energy delivering capacity. But it may well be less expensive than putting down additional rotary machine substations. The method has led to the existence of such "copper mines" as bunches of three-wire lines of 1 in.^{2} outers in some densely loaded areas. To-day the mercury arc rectifier permits of the placing of smaller substations directly upon the distributing network, which is more economical than running heavy feeders from the original substations. That plan has found favour in some areas of the residential class where the load density is moderate and the distances from the substations long, so that increased loading by the adoption of electric cooking, etc., has produced excessive voltage drop in networks which served quite well for lighting loads. The addition of rectifier substations involves the laying of h.t. feeders for their supply, unless the original h.t. network serves.

It will be worth while to consider whether such reinforcement is likely to suffice for a reasonable future, or whether it would not be better to go the whole way and convert to an a.c. system. It can be said for the rectifier as a temporary provision that the substation and h.t. feeder will serve equally for the transformers when conversion is decided upon, so that the cost of the interim step will only be that of the rectifiers and their accessories, which may not be entirely sacrified when conversion takes place. One rectifier set may serve at a number of places in succession when conversion is in progress over a large area of supply.

If the load density along a distributor has become locally

excessive it may be less expensive to supplement the loaded length by additional copper than to put down a new feeder from the nearest substation. The additional copper will be more economically added in the form of cables parallel with the old ones, transferring some services from the old to the new, than by putting in a new heavier cable and taking out the old one. A cable which has been cut into and tapped at short intervals has only scrap value ofter removal. Changing the cable will involve more interruption to consumers' supplies than transferring only some of the services from the old to the new cable. If a spare duct has been laid alongside the original cable, the laying of the new one will be greatly facilitated and cheapened.

There is a clear distinction between reinforcing an existing d.c. system and extending it. There may still be some cases where extension is required. If the voltage drop limit will be exceeded, probably the mercury-arc rectifier will afford the cheapest way of meeting the requirements, laying four-wire cables to be used as three-wire until a.c. conversion is carried out. Then only alterations to some of the services will be necessary. But it would probably be better still to cut out the intermediate stage and put down a three-phase transformer substation at once, so saving the exchange of consumers' appliances, meters, etc., eventually.

For the reinforcement of an existing a.c. system, almost always the most economical plan is to feed it at more frequent intervals from additional substations. If the need has been foreseen in the lay-out of the original h.t. system, or a spare duct has been laid alongside the l.t. mains, the cost should be reasonable. This matter has already been discussed in the chapter on system lay-out and networks. Exceptions may arise in the case of long dead-ended mains, not heavily loaded but having an excessive voltage drop at peak loads on account of the distance from the feeding point. Such cases may be dealt with by the use of some form of voltage regulator, usually one of the automatic type requiring no attendance. Other methods may be feasible, for example, cross-connecting the ends of two mains to make a ring or loop will help if one has a smaller voltage drop than the other; or if there is a difference in the time incidence of the peak loads in the two. In exaggerated cases of this kind it may be well to consider substituting a h.t. feeder and putting transformers on it for some or all of

the individual loads. Evidently it would have been better to have put down the h.t. feeder originally ; but one cannot always foresee what will happen along suburban roads which looked like remaining open highways indefinitely.

There are many areas where the distribution is by three-wire single-phase mains with 200 volts across the outers. If the cables are sound, the voltage can be doubled. Experience shows that many three-wire concentric cables for 100–100 volts laid many years ago are quite reliable for 200–200 volts. The change means replacing all consumers' appliances, meters, etc., but even so it is a cheap way of quadrupling the load capacity of the cables. They can be fed from the centre-tapped 400-volt secondaries of three-phase transformers, which can be used later for four-wire distribution, by a simple change of the connections. They cannot be fed as 230–230-volt three-wire lines from such transformers, because the two outer currents add vectorially in the middle wire and will overload it, with heavy voltage drop.

More serious considerations arise when the primary h.t. feeders to any area become or threaten to become overloaded, unless the contingency had been provided for in the original lay-out, e.g. by running spare ducts along the original h.t. routes. Some relief may be obtained by cross-connecting the ends of adjacent feeders into loops, particularly if advantage is taken of some diversity in the time incidence of the individual peak loads. For more ample reinforcement there are the alternatives of running additional cables or laying a new e.h.t. ring or rings, say at 33 kV., with step-down transformers to feed the old primary at convenient points. Raising the voltage of the old primary will rarely be practicable unless the cables were chosen with that step in view. Cables rated for 6·6 kV. sometimes prove safe for 11 kV., but it is hazardous to try that on cables which have been in use for several years. Also the actual job of changing over the transformers presents many difficulties. On an overhead system the raising of the line voltage is practicable if the insulators were chosen to be suitable for the higher voltage, or if they can be changed readily. That has been done in some cases.

The superimposition of an e.h.t. system has much to recommend it as a method of reinforcing a large system at minimum cost. It reduces the total current leaving the source and the difficulties arising from the bunching of feeders in its vicinity.

The percentage voltage drop in the primary system is smaller. The old primary network, fed at frequent intervals, will have its efficiency and capacity increased. Where a d.c. distribution has been fed from rotary substations, those substations will be good places for the step-down transformers. A ring or rings will usually be the preferable form of lay-out, affording two-way feed to all substations with facilities for sectionalizing.

Unless there are spare ways or ducts on suitable routes, it will be best to plan the routes of the e.h.t. lines with regard only to the positions of the step-down substations.

Where rotary machine substations serve tramway systems as well as the general distributors so that some of the rotaries must be retained, the converter transformers should be fed from the stepped-down voltage, not changed for feeding from the new e.h.t. lines. The step-down transformers used to supply the traction load as well as the general load will be used at a better load factor than if the converter transformers were also supplied from the e.h.t. lines, and the cost of replacing the latter will be saved.

When the problem is that of the extension of an existing area of supply into a new one, it will usually be best to plan the new distribution as if for an independent area, taking the points on the existing primary system whence the new one will be fed as the source or sources, the starting points for the new primary. The neighbourhood of a C.E.B. line or substation may provide a more economical source than the existing system; or both that and the existing primary may be advantageously utilized. All the local circumstances must be considered. The initial load and its rate of growth will usually be estimated. Where the designer who considers too intently the initial cost of extension may be caught is by extending a primary system working at a moderate voltage—say 6·6 kV.— beyond its economic radius on the assumption that the new load will be only small for some years to come, and then finding within two or three years that the 6·6 kV. lines are overloaded. Where there is even a bare chance of that it will be wiser to put down cables for a higher voltage, e.g. 33 kV. They can be worked at 6·6 kV., i.e. as an extension of the original system, whilst the loading permits; or a step-up transformer may be put in at a suitable point; eventually a 33 kV. feeder must be run from the source across the original area of supply, when the load justifies that.

In this country extensions of the kind under discussion will be into rural or semi-rural districts, or into "satellite" towns or the like. Examples of the former class are the rural areas around Bedford and Norwich, where most of the lines are overhead; but in the Norwich case an 11 kV. cable ring serves part of the area.

It is likely that the C.E.B. grid will make 33 kV. available for many such areas and that 33 kV. will become a very general primary voltage.

SUBSTATIONS: TYPES AND APPLICABILITY

CONTINUOUS CURRENT SYSTEMS

SUBSTATIONS for continuous current (or direct-current) systems are equipped with converting machinery which may be conveniently classed as—

(*a*) *Rotary*, including rotary converters, motor-generators, motor-converters; and,

(*b*) *Rectifiers*, of which at present only the mercury-arc type need be considered.

(*a*) Of the rotary machines the rotary converter has the highest efficiency *per se* and the lowest cost per kilowatt capacity; it has to be supplied through transformers. The motor-generator needs no transformers if the supply voltage is not over about 11 kV.; it is the most costly of the three and has the lowest intrinsic efficiency. The motor-converter is intermediate in cost and efficiency between the other two. It requires transformers. The rotary converter has the characteristics of a synchronous motor, that is, it needs synchronizing gear for starting and runs at synchronous speed independently of the load. The output voltage is controlled on the a.c. input side, not by excitation. It is possible to obtain some power factor correction by means of regulation of the field, which is sometimes an advantage; although such correction involves a sacrifice of d.c. output. In other words the maximum capacity is a kilovolt-ampere input capacity.

The other two types are susceptible of voltage control by excitation. If the motor element of the motor generator is of the induction type, as is usual, it does not need synchronizing at starting and its speed will vary inversely (but not proportionately) with the load.

The motor-converter is a synchronous machine susceptible of d.c. voltage control by excitation.

All three types have a no-load or minimum rate of loss in addition to load losses, of considerably larger magnitude than transformers. As a general rule rotary substations are attended; but of recent years remote control and automatic control equipments have been developed and used, by which

the machines can be started, stopped, and regulated from a control centre, or automatically, in accordance with the load variations.

(*b*) The mercury arc rectifier has found wide application in recent years. It requires transformers ; and the d.c. voltage can only be regulated on the a.c. input, unless the rather novel method of grid control is used. Either can be made automatic. The rectifier is available in comparatively small sizes and occupies but little space, for instance it has been installed in under-street chambers. Constant attendance is not necessary ; remote control is frequently applied. It has a voltage drop of the order of 15–20 volts independent of the load ; hence its intrinsic efficiency increases as the d.c. output voltage is raised. Up to some capacity which it is difficult to express (as makers claim increasing ranges), the glass or quartz globe type is satisfactory, i.e. it has a reasonably long life of several years. Large capacity rectifier units are of the steel-encased type. They require vacuum pumps and cooling water circulation, and are therefore intrinsically more expensive than the glass or quartz type ; but a longer life is claimed and seems probable. If units above some maximum capacity are required the steel-enclosed type must be adopted: just where the dividing line comes when all factors are taken into account can only be ascertained by getting tenders to specifications. The mercury-arc rectifier is suitable for relatively small sized substations distributed over a d.c. network, and for electric traction d.c. systems.

The rotary machine substation has to be of fairly large size to keep the capital cost per kilowatt low, and to be run at a good load factor to make the overall cost per kilowatt-hour low. It is therefore suitable only for densely loaded areas. The mercury-arc substation can be much smaller without involving excessive cost per kilowatt ; it can be used to increase the capacity of a d.c. network which has become overloaded by putting down rectifiers at several places and so reducing the loading and radius of distribution. Such a use has been made of it on suburban networks originally supplied from large rotary machine substations, in preference to the more expensive course of conversion to a.c. distribution ; perhaps as a prelude to the conversion.

It is unlikely that d.c. distribution will be adopted for new urban areas, or even for considerable extensions of existing systems.

Conversion for traction supply will have to be considered later as a special case. Conversion for consumers who must have d.c. for their particular purposes hardly comes under the subject of distribution.

It may be noted that all types of converters have no-load losses of greater magnitude than those of transformers; and that the energy delivered through them has to carry the capital charges on the converters, which are much heavier than those on transformers.

Batteries. A fairly common adjunct to rotary converting plant is a storage battery. Economically a battery may earn its keep if it can carry the light loads for a few hours of the night, or take enough of the peak loads to avoid running up a converter set. In the past, storage batteries were used for such purposes, as well as for emergencies when the running machines were put out of action for any cause. To-day the magnitude of night loads and peak loads makes such uses impracticable. A battery is in itself a costly equipment; it takes up a large space which must be adopted by ventilation, and acid-proof construction, etc., so that a spare converting set is a cheaper stand-by. A Diesel engine set is often a more economical peak load and emergency provision. A battery has heavy losses: from charge to discharge the energy loss is of the order of 30 per cent. Hence a battery is generally out of the running as a part of the load carrying equipment. But there are some subsidiary services, such as power for operating switchgear or some small auxiliaries, and emergency lighting of the substation which may justify the installation of a small battery. A radical difference between a battery and a generating or converting set is that the battery has only a limited ampere-hour capacity; whereas the others will give their output for as long as needed.

A battery may be justified in a traction converting substation to even out a "peaky" load.

ALTERNATING CURRENT SYSTEMS

Transformer Substations. The most economical location and sizes of transformer substations to supply a given area could be calculated if all the relevant data were available; which is rarely the case. As the predominant factors in distribution costs are the charges and losses pertaining to the l.t. mains, some approximation can be made by considering a hypothetical case resembling as nearly as possible the kind of load distribution

to be expected. Supposing that a length of distributing main has a uniformly distributed series of loads along it, the most economical current density for the kind of cable to be used is calculable, and so the voltage drop per 100 yds. That sets a limit to the spacing of the substations along the length, which is not necessarily the most economical spacing. For the same load distribution a cable of half the section and transformers at half the spacing would give the same current density with one-quarter of the losses and half the voltage drop. There would be two substations each of half the capacity of the former one. The two would cost more than the one of double size; there would also possibly be an additional length of h.t. main to supply the two. The sum of the capital charges and the value of the annual losses in each case would show which was the more economical. A few trial calculations of this kind will show that there is some spacing below which the extra costs and charges on the transformer stations and h.t. line exceed the saving on the l.t. charges and losses. So the allowable voltage variation gives an upper limit for the spacing, the balancing point between l.t. savings and transformer and h.t. charges a lower limit.

In densely loaded areas several hundred kilowatts capacity may have to be put into one substation, requiring a special building and the acquisition of a site for it. Such an area is likely to be one where sites are expensive; so that it may prove less expensive to find four sites to accommodate, say, 250 kW. each, than one site to accommodate 1 000 kW.; although the transformers themselves and the h.t. feeds to the four may be considerably more expensive than the larger substation. There will, however, be some saving on either the l.t. mains or the losses in them if advantage is taken of the shorter radius of distribution. This is an example of the kind of practical circumstance which modifies theoretical calculations; it does not make them valueless, however. In planning for such areas, besides the relative cost of larger and smaller transformers, and sites and buildings, the relative costs of taking l.t. feeders to the one site, and of bringing h.t. branches to the several sites, have to be considered. These questions arise particularly in cases of conversion from d.c. to a.c. Usually there will be a converting substation in being which will accommodate much more transformer than converter capacity. But the d.c. distribution with heavy feeders to points on the network is not

a good model for a.c. distribution. The far ends of those feeders may probably be good places for transformer substations dividing the load between them. Also there are likely to be some large consumers who can be more economically supplied by transformers on or near their premises. That will relieve the load on the l.t. distributors. The old converting substation will probably have been served by e.h.t. feeders; or can be readily brought into such a primary system, whether old or new. It is then available for a primary to secondary step-down and switching station.

Special accommodation is always necessary for step-down stations, say from 33 kV. to 6·6 or 11 kV. Since the economic feeding distance with either of those pressures is much greater than with the d.c. l.t. pressure, the step-down capacity can be much greater than the former converting capacity. Hence some valuable sites which formerly served for converter stations may be disposed of; a credit to the conversion account.

In a new system it will rarely be necessary to site step-down stations where land is dear or covered with buildings. E.h.t. switchgear, ventilating fans, oil tanks and pumps, and other auxiliaries of a step-down station take up a good deal of room. Typical lay-outs for a given ultimate capacity will give the ground or floor area required. There is considerable working advantage in having everything on one level. An outdoor lay-out of switchgear and transformers may prove more economical than one in a building. Control gear, instruments, meters, etc., can be housed in a comparatively small building. The choice between outdoor and indoor, or between all on one floor and in several storys, is largely determined by the cost of land and buildings.

Distributing substations of moderate capacity may be underground chambers. Probably 100 kVA. is about the practicable limit for chambers under the streets. The Commissioners' Regulations put it at 75 kVA., but permission may be given for larger capacities on cause shown. The permissible sizes of manhole covers and the facility of handling probably make the largest convenient size of transformer about 25 kVA. Three of that size make up the Regulation maximum. Adequate ventilation may usually be obtained by up- and down-cast air ducts, the motive power being the heat from the transformers. Road authorities are inclined to object to large under-street chambers; they may be over-ruled on appeal to the

Commissioners, but such a procedure does not make for comfortable relations.

Larger accommodation may be found on plots of small commercial value, enough perhaps for outdoor transformers; occasionally one may find a suitable building with a roomy basement, or erect such a building and let out the upper parts. The freehold should be secured in either case.

Under-street chambers will often prove of sufficient capacity in not very heavily loaded urban areas. In residential suburbs, for example, it may be a good plan to provide such pits at street corners to feed a radius of, say, a quarter of a mile; i.e. the distances between the pits to average half a mile. They need not all be equipped, or even built, at first, but means of reaching each site by h.t. branches should be provided concurrently with the laying of the l.t. mains. Such a plan lends itself to the use of a uniform size of distributing cable, and minimizes the problem of voltage regulation.

For rural districts with overhead distribution, pole-mounted transformers will serve for loads of up to about 50 kVA. For larger loads—as that of a compact village—the outdoor substation within an enclosure is suitable. It may be noticed that 25 kVA. is about the lower limit of economical transformer size for supply from a 33 kV. line. (This may not be true very soon.) As such an arrangement eliminates one stage of transformation, there are savings over the alternative of supply from the general 11 kV. lines. Obviously such savings will not pay for more than a short extension of the 33 kV. line.

Separate transformers are indicated for consumers with loads of hundreds of kilowatts anywhere or in rural areas where the consumers are scattered so that a l.t. network is impracticably costly; and in some other conditions, e.g.—

(1) In densely loaded areas where substations and l.t. mains are already heavily loaded, and increase of capacity is very expensive.

(2) Where the consumer requires d.c. in such amount as to make a converter from the h.t. mains economical.

(3) In "thin" areas where houses are widely spaced, or stand well back from the road so that services are long. It may be more economical to serve such areas by h.t. mains and to put transformers in the houses, than to lay l.t. mains, and long services. This is a common practice in suburban areas in the U.S.A. The conditions approach those of rural distribution.

In this country such transformers and the h.t. gear and connections must be enclosed, accessible only to the staff of the supply undertakers; and must comply with other safety provisions laid down by the Regulations. Where the distribution is by overhead lines, the pole-mounted transformer is indicated.

It may be worth while to point out that the provision of transformers for individual consumers, involves a larger total transformer capacity, more cost per kilovolt-ampere, and heavier magnetizing current losses. Each transformer must be capable of carrying the maximum load of the consumer; so there is no diversity factor in operation.

This illustrates the fact put in a different way above, that l.t. costs and losses may be reduced without limit to zero by multiplying the transforming points, but with some enhanced costs and losses elsewhere.

SUBSTATION BUILDINGS, ETC.

Transformer Substations. The modern oil-immersed steel-tank static transformer has no intrinsic need for the protection of a building. Outdoor types are quite common from the pole transformer of, say, 25 kVA., up to the giants of 50 000 kVA. and more of the Grid and power-houses. Distribution transformers on underground systems in built-up districts cannot be allowed the ground space necessary for the fencing of the outdoor type; some form of enclosure is necessary. The simplest and cheapest forms are either the kiosk or the underground chamber. The kiosk may be an iron or steel box, usually with a separate switch compartment, or a brick or concrete hut, according to the capacity desired. Small sizes may be located on footpaths, if the road authority consents; larger ones require more space which must be found off the street. Several makers have standardized the metal box type with suitable switchgear at reasonable prices for up to about 50 kW. transformers. Brick and concrete buildings are used for capacities up to about 500 kVA.: they have switchgear compartments, sometimes controlled from a centre; and frequently ventilating fans which come into operation when the transformer load reaches a certain point.

This kind of substation building offers opportunity for design to meet particular requirements on such sites as may be available; it is adaptable to the requirements of a switching

point on the high-tension system, to prospective growth of load, and to housing an attendant.

From the point of view of economic engineering it is better to design the structure as a containing shell for the apparatus it is to shelter than to shape it internally to serve as part of the supports of the switchgear, and so on. There must be ample space to get round all the apparatus for inspection.

Reinforced concrete will usually be the cheapest material for walls and roof. An efficient damp course and waterproof floor are essential. Architectural adornment has no economic value; if authorities require the building to look like something quite alien to its purposes the luxury must be paid for by someone.

The under-street transformer chamber is the cheapest in built-up districts. It entails no purchase of site; in this country authorized distributors have an established right to construct it. It can be built on the line of the mains at such strategic points as street corners, feeding in several directions with a minimum length of idle mains; and it is eminently adaptable to meeting the growth of load by feeding the l.t. mains at shorter intervals.

There can rarely be any choice of material for floor and walls outside concrete; the roof a grid of rolled I-joists filled in with reinforced concrete, with a frame for the cover to the manhole. Brickwork laid in cement mortar makes as good a job, but will usually be more expensive. Ventilation by downcast and upcast earthenware pipes opening into grated traps is usually sufficient for the cooling of the transformers. Walls and floor should be watertight; if the soil is saturated or water-bearing, an asphalt sheet may be incorporated into them. The floor should drain to a sump into which a suction hose of a pump can be lowered. Flooding from heavy rain, or from the bursting of a neighbouring water main should be provided for by putting all switchgear, fuses, etc., into watertight boxes; then flooding will not matter. No high-tension switchgear is permitted in under-street chambers in this country.

Substations on e.h.t. systems, whether for step-down transformation or for switching only, must be above ground, practically always on acquired sites. If ample space is cheaply available, outdoor type switchgear and transformers will save the cost of buildings. There must be some shelter for control gear, instruments, etc. What has been written above in regard to the hut form of transformer kiosk applies here. In built-up

districts where there are no vacant sites, it is sometimes possible to acquire the basement of a building and convert that into a substation of either kind. Where the building belongs to a large consumer such an arrangement may be made as part of the agreement about terms of supply. Leasing such accommodation is objectionable; the safe policy is to acquire the freehold of a building and either adapt it or rebuild, letting off any portions not required.

Rotary Converter Substations. The requirements of accommodation for substations on d.c. systems employing rotary machines are much more exacting than those of a.c. substations. There is not much choice of location; the substation must be somewhere near the load centre of the area served. The ground area must provide a machine room floor large enough to permit all-round access to the machines, and of lifting out and setting down of any part for inspection or overhaul. The height must allow for a crane for the purpose.

The low-tension switchgear should be in the machine room so that the heavy conductors may be short, and so that circuit-breakers and instruments may be in sight of the machine attendants. High-tension switchgear is better outside the machine room : it may be of the outdoor type if the site permits. The transformers must be fairly close to the machines, preferably outside the machine room—along one side wall under a shelter roof, for example. If a battery is to be installed, a separate well ventilated and lit room is needed.

As in other cases, from the aspect of economic engineering it is best to consider the building as a shelter for the equipment. For example, the crane gantry will be more economically supported on steel columns than on the side walls. If the site allows freedom in the lay-out it should be remembered that the cost of roofing goes up rapidly with increase of span; a building twice as long as its width will cost less to roof than a square building of the same floor area. Of course there are limits to this; but it has a bearing upon putting the transformer and high-tension switchgear outside the building.

There must be easy access from the street for the largest transformers and machine parts, good ventilation, and good natural lighting if possible. The transmission of noise and vibration to neighbouring buildings must be minimized; ample cable-ways from the street mains to the switchgear must be provided.

Probably there will be few new converter stations built in this country outside those for traction purposes. If the need does arise, local conditions will be the determining factors in the style, material, and ultimate cost of the buildings. There is no compensation in earning power or efficiency for costly ornamental features, but ample return for forethought and expense in making the buildings convenient for their purpose.

Many existing rotary machine substations are housed in buildings which were formerly generating stations. Strictly, the remaining capital charges on the buildings should now be debited to distribution.

Rectifier Substations. Rectifiers are more easily accommodated as regards sites and buildings. They are of light weight requiring no heavy foundations, quiet and free from vibration, and occupy but little space. Perhaps the main point in their favour is that they can be of small individual capacity without being unduly expensive, which favours spacing out rectifiers along distribution routes instead of at centralized stations.

Rectifiers of up to 50 kW. with transformers and switchgear can be lodged in under-street chambers. More room is necessary than for transformers of equivalent capacity ; more ventilation, so a fan is needed. Making allowances for these extras, what has been written above about transformer substations is also applicable to the rectifier type. For general distribution the rectifier seems likely to have but a transient existence. Traction applications will be dealt with in their place.

Evidently any substation which requires personal attendance must have some minimum accommodation for the attendants, which adds to the cost of the buildings. That is one of the items to be set against the cost of remote control apparatus when considering the pros and cons of that method of operation.

ARRANGEMENTS FOR LARGE CONSUMERS AND
SPECIAL REQUIREMENTS

CONSUMERS with loads running into hundreds of kilowatts should not be supplied from general low-tension mains, but from substations on or closely adjacent to, their own premises. If on the consumers' premises, high-tension services, switchgear, and transformers have to be safeguarded by precautions, including limitation of access to the employees of the supply authority; or if the consumer requires high-tension within the premises, then the works become subject to the Factories and Workshops Special Regulations. It is a moot point whether those Regulations apply to high-tension gear on the consumers' premises to which only the supply personnel has access. The Home Office claims that such substations are "factories," but will exempt them from the Regulations applying to high-tension work accessible to the consumers' workpeople. The Factory Inspectors may, and do, make suggestions for safety measures in excess of those required in the undertakers' own substations; and have power to institute proceedings where they consider that the undertaker is in default. This is only mentioned by way of caution; it is not a complete statement of the legal responsibilities. Extra-high-tension supply to a consumer (over 3 000 volts) requires special consent from the Commissioners.

In these cases of high-tension supply, the consumer may agree to pay for the supply at high-tension rates, providing his own transformers, etc. The undertakers' expenditure is then limited to the high-tension line and switchgear meters and protective gear. It is rather a technical than an economic point that if the consumer uses motors and the like, at the h.t. supply voltage (e.g. 6·6 kV.), his wiring to such appliances becomes part of the h.t. system; the protective gear should secure the rest of the system against faults and overloads arising on that wiring. The cost of these provisions should be considered in the charges made for the supplies. Sometimes supplies of the kind are taken as stand-by or peak-load provision to supplement the consumers' own generating plant.

Obviously the charge made should include a proper sum for the capital charges involved; independent of the use made of the connection.

It is always advisable, and may be essential, to have alternative routes of supply to large consumers. If special feeders have to be laid to serve them, the most economical sizes of cables and other elements of cost become worthy of attention. The load and form factors of the load will usually be foreknown with fair accuracy; they may result in a current density different from that in the general system. In districts mainly residential, comparatively small factories may be most economically served from special substations. If the new load would be excessive for the existing distributors, it is a question whether to reinforce them, or to extend the nearest high-tension main and put in another transformer.

Large buildings such as hotels, office blocks, department stores, blocks of residential flats, etc., offer some diversity of conditions. Where the proprietor is to be the sole consumer, as in the case of hotels, a substation in the basement of the building is usually practicable. Where there will be many consumers, the individual tenants of flats, and so on, the owner may expect a rent for accommodating the substation. The rising and floor mains will be virtually part of the undertakers' distribution system, although they may be provided by the proprietors of the building. As the losses in those mains are part of the distribution losses, the undertakers have a real interest in seeing that they are of adequate size for the load. The proprietor may not wish to spend so much as the undertakers would like; the latter have to bear the brunt of complaints of low voltage from the consumers as well as the value of the losses. It is evidently a matter for negotiation with the proprietor initiated at the earliest opportunity after it is known that a building of this class is to be erected. The architects and builders can do much to facilitate or render irksome the connection of individual consumers, the placing of their meters, etc., and touch should be kept with them throughout the planning and erection. From some points of view it would be better for the undertakers to own the mains throughout the buildings. Prior settlement of these matters is preferable to raising prickly three-cornered questions between consumers, proprietor, and undertakers afterwards. The undertakers are in a good position for negotiation, as the proprietors must offer

their tenants facilities for supply; the undertakers are not obliged to do more than put in a service and meter for the whole building, leaving the proprietors to make their own arrangements for retailing to their tenants.

In the "sky-scrapers" of American cities it is now common to find high-tension rising mains to substations on several floors, saving much copper and voltage drop over the alternative of heavy low-tension risers. The facts that these buildings often house as many people as a small town, and that the American low-tension supply is at 115–230 volts give more powerful inducements for such arrangements than are likely to occur in this country.

It is well to arrange that the transformers for large buildings also feed into the general distribution system outside it. That will improve their utilization, particularly desirable in the case of an office block which is likely to have, by itself, a rather poor load factor.

Where large buildings stand on an island site it is a good plan to put down one substation in the street on each frontage and take in as many l.t. services feeding into a ring main inside the building, whence the rising mains may take off at convenient places, as convenient service entries and rising main positions may not match. Such an arrangement removes possible difficulties about access. The outside substations may be switched out one at a time for inspection, change of transformers, etc. It may be possible to put the transformers upon different h.t. feeders, giving alternative routes of supply; but in that case it will be necessary to ensure that in the event of one feeder being switched out for any purpose, it is not kept alive by paralleling through the l.t. sides of the transformers. Probably some of the most awkward cases of supply arise where old large houses are converted into blocks of flats. It can only be repeated that early conference with proprietors, architects and the rest, is essential for arranging satisfactory service. Ring mains on each floor are desirable. Rising mains of bare copper on insulators in special shafts may be less costly than cables.

A requirement occasionally met with is the provision of emergency service independent of the main supply. That obtains in theatres in London. (It is obviously desirable to avoid complete darkness in places of public assembly.) The requirements can rarely be met to the extent of giving supplies

from different sources. It may be that supply from different feeders will be accepted. In the London theatre centre there is a d.c. system fed from Diesel engines which is a very satisfactory way of giving the desired "police" lighting in supplement to the main a.c. supply. It is rather expensive, hardly practicable as an economic resource in other than exceptional circumstances. More practicable perhaps is the installation of an accumulator, trickle charged from a rectifier; with automatic switching of the "police" lighting in case of interruption of the main supply. Generally such provision is a matter for the consumer; the undertaker can help by giving a low price for the charging energy. Such battery stand-bys are particularly indicated for surgical operating theatres; again at the consumer's cost.

Particular Needs of certain Consumers. Some of these are: continuous current for particular operations; and highly fluctuating loads such as those of arc furnaces, welding machines, rolling mills, etc., high-frequency furnaces and tool drives. As a general rule, all kinds of converting machinery whether for rectifying, change of voltage, frequency, and number of phases, should be provided by the consumer. The undertakers' interest is to see that what is installed will not injuriously affect voltage regulation and other conditions of supply to other consumers. For example, the starting current of large motors needs limitation; arc furnaces and welding machines give a very peaky load. It is advisable to persuade consumers who propose to use any of these things to specify performances and appliances which will minimize load fluctuations.

A common feature of industrial loads is poor power factor. A rational tariff will offer the consumer an incentive to keep his aggregate power factor reasonably high. Making him pay for poor power factor does not remedy the inconveniences of voltage drop or high mains losses. The undertaker may have to do some power factor correction in his own substations; the consumers who cause the trouble should pay for the remedy.

Where fluctuations of load and poor power factor prevail, it is advisable to supply the works concerned from separate transformers. A connection from the general distributors may be provided for part of the supply, e.g. the lighting.

In this country an authorized undertaker is entitled to refuse to connect to consumers appliances which will injuriously

affect the service to others, or the undertakers' property. But the limits of reasonable cause for refusal are not well defined; it is better to agree with consumers upon what they may put in and how they will use it than to engage in disputes.

The development of rectifiers of various kinds—copper-oxide, valve, mercury arc—has greatly facilitated the supply of d.c. for loads of moderate and small magnitude. For example, an electro-plating shop can be equipped with a rectifier of suitable capacity and voltage for each vat at a much smaller cost than that of rotary machines; they will need no attendance, oil, brushes, etc., and be generally more efficient. Electro-chemical operations requiring large currents at low voltage will doubtless continue to be served by rotary converters (until metal rectifiers become available), they can be designed to provide some degree of power factor correction for other loads. The way in which single-phase appliances, such as welders, are supplied from three-phase mains may produce serious phase unbalance, or none at all; though it is bound to produce a "peaky" load of low power factor unless corrected. Obviously a powerful form of persuasion is a method of charging which favours the consumer whose power factor and phase balance are good. It is most important to develop such relations with large consumers that they will spontaneously consult the undertakers' engineer before they decide what conversion and transformation plant they will install. The undertakers' "consumers' engineer" should be well up to date in this field.

The "electrode boiler" is coming into some favour as a means of central heating. It is supplied at high voltage— 6·6 kV. or more—with the neutral connected to the boiler shell. This infringes the regulations about earthing, but has been permitted in some cases. Apart from that it is advisable to consider whether the possible irregular working of the boiler is likely to have any undesirable effects upon the rest of the system. A separate transformer seems a more comfortable arrangement than a direct connection.

There are a few instances, likely to be added to, of very large works being supplied by special e.h.t. feeders from generating and grid stations. In some of these cases the works have had their own power stations which may or may not be kept available for running in parallel with the supply system. These cases are always subject to special agreements which

should be quite definite about the magnitude, load factor and power factor of the load ; the calculation of the most economical transmission lay-out is then simplified.   .

Factory Acts Regulations.   The new Factory Act which becomes operative on 1st July, 1938, clarifies and extends the definitions of the premises to which " Regulations for the use of Electrical Energy " will be applicable.   Doubtless a new set of Regulations will be issued shortly, and should be studied. The Regulations will interest supply undertakers as well as consumers.

NON-STANDARD SYSTEMS FOR SPECIAL CASES

Street Lighting. When street lighting is adopted along routes where distributing mains already exist, the street lamps can be supplied from those mains. The questions which then arise are mainly concerned with switching on and off. If a switch wire has been provided with the mains, there will be little reason to do otherwise than use it, with a magnetic relay in each lamp equipment. Time switches are frequently used; they may have an "astronomical" feature which controls the switching times by the calendar. Recently there have been some trials of photo-electric relays which have the advantage of responding automatically to variations of natural light. Both types require some attention and maintenance. The initial cost, reliability, and maintenance cost of time switches are well established; whether that can yet be said for the photo-electric type is doubtful. Information should be obtainable from both users and manufacturers before long. A few instances are on record of the use of valve relays actuated by a high frequency current imposed upon the mains. This plan is said by users to be quite satisfactory; experience is at present limited. Any of these switching equipments adds considerably to the cost of the apparatus at each lamp-post. Probably the magnetic switches and a switching line will give the minimum total annual cost, unless the running of a switching line involves breaking up and reinstating paving.

Time switches are essentially clock movements, a kind of mechanism one would rather not put into lamp-posts or lanterns. With a switching line a time switch in a substation can control a large number of lamps; and is in a more favourable environment for good performance and examination at intervals; whilst a solenoid-actuated mercury switch at each lamp-post is a fairly robust and comparatively cheap mechanism.

When special mains have to be laid for street lighting it is worth while to consider whether a parallel or a series system should be used. The series system has some advantages. Only one conductor is required for any line of lamps which forms a

loop. For instance the line can go out on one side of the street and return by the other feeding the lamps on both sides; it can pick up lamps on side streets and up to long distances on any irregularly scattered plan. There will be no voltage variation between the nearest and the most remote lamps. The conductor can be of smaller copper section than the double conductor required for a parallel system.

Against these advantages the line must be fed through a constant current transformer—an appliance of respectable antiquity and proved reliability—the cable must be better insulated than for a parallel system, and a safety cut-out must be provided at each lamp-post to prevent the circuit being opened by a lamp failure. Probably the best arrangement on a series circuit is to put a transformer at each post so that the lamps and holders are not at the comparatively high line voltage. The series plan with transformers has been extensively used for many years in the U.S.A., at first for arc lamps, later for filament lamps; recently for some of the new gaseous discharge lamps. Standard series transformers as used in the U.S.A. will be produced here cheaply if there is a demand for them. Transformers can evidently be used on parallel circuits; they permit the use of lamps of other than the general distribution voltage, and are less liable to give trouble from leakage than lamp fittings on the general mains. It seems likely that gaseous discharge lamps will become generally used for street lighting, when it will be an advantage not to be tied to a particular voltage.

Since street lighting is usually run from dusk to dawn it has an annual load factor approximating to 50 per cent. The economical current density for that load factor is about 1·4 times that for 100 per cent load factor. On a series system the conductor is of uniform section all round the circuit, so it is easy to select a cable size approximating to the most economical density with no restrictions of voltage drop. On a parallel system the voltage at every lamp may be equalized by running the two-conductor cable as a loop, with both ends brought to the point of supply and one of each pair of leads connected to the transformer as shown in the diagram on the next page.

The voltage drop is then the same to every lamp and need not be considered in selecting the conductor size which may be uniform all round the loop, or stepped down at each lamp. Either is less economical than parallel connection, because the

total current circulates round the whole length of the loop. With uniform conductors and ten equidistant lamps the losses are $3\frac{1}{2}$ times as great for the single end connection as for connecting both ends of each lead. It may sometimes be a useful arrangement on long roads where only a low tension supply is available at only one end, a series system would attain the same result of even voltage more economically if a suitable supply were available.
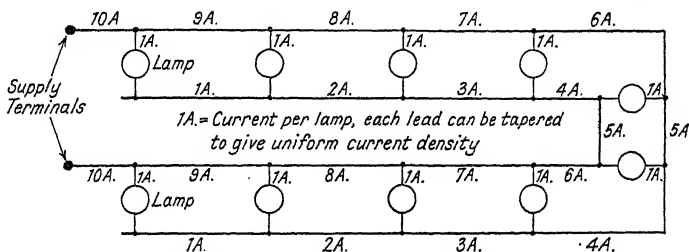
Fig. 6. Equi-voltage Loop for Street Lighting

Improved road lighting is very much in demand to-day; and requirements for lighting long stretches of main roads are likely to arise where economy in mains and the need for even voltage may both be best served by a series system. A series system is sometimes quite useful for the general lighting of large halls, or for the temporary lighting of outdoor shows and the like with lamps or groups of lamps of high candle-power— a reversion in principle to the old series arc circuits.

Electric Traction. Distribution for electric traction is a subject which might occupy a substantial volume for complete description and discussion. Only some salient points of economic design can be touched upon here.

In some respects the problem of economical design is simpler than that for general distribution. The location and magnitudes of the loads are foreknown from the prospective service; the weights and speeds of cars or trains; the gradients and stopping places or stations; the characteristics of the motors to be employed; the points of supply; and the voltage to be used on the vehicles. Given the working time-table and plan and contour of the several sections of the line, a load plan can be plotted for each section for several different times of day (corresponding to the services to be run at such times) and daily load diagrams made out for different seasons of the year.

Hence a very fair forecast can be made of maximum loads, of load factor for each section and for the whole line. There is much less uncertainty about what has to be provided for; therefore economic calculations have a smaller element of speculation than in the general distribution case.

Another favourable point is that the voltage variation with load may be considerably greater than is permissible in general distribution without serious detriment to the service. Of course, the prospective time-table may not be adhered to in practice; usually there will be some positive limitation of the maximum number of trains or cars which can be put upon each section.

In what follows it will be assumed that the load plans have been worked out for the primary supply to the transport lines, resulting in a plan of substation positions of certain capacities and of a suitable primary lay-out of mains. These may be part of a general transmission system or special to the traction undertaking according to circumstances, calculated on economic principles as for any other class of distribution, but taking advantage of the greater latitude of voltage variation which is possible in the traction system. It seems that in any new traction work requiring continuous current supply, the sub-station equipment will be of the rectifier type, reducing the l.t. feeders to a minimum, usually confined to the routes of the traction system between the substations and the feeder pillars or their equivalents. It may be observed that converting plant cannot be used simultaneously for general and traction supplies so that there will usually be little advantage in design-ing substations to serve both purposes if that involves laying l.t. feeders of appreciable length to the traction route. The existing facilities of remote control and automatic control of converter substations eliminate the expensive continuous attendance which in the past has militated against the adoption of relatively small substations. There is nothing against utilizing a common h.t. network for the general and traction supply : usually there will be some advantage arising from the diversity factor between the two classes of load.

Tramways. The standard tramway with return conductors by the running rails is an unsymmetric electrical system as regards potential to earth of the go and return lines. In this country there is a regulation limiting the potential to earth of the rails to 7 volts ; there are similar regulations elsewhere. Even where there are no regulations there are good reasons for

limiting the potential and magnitude of the "stray" currents which return by other paths. These limitations may impose the necessity of supplementing the conductivity of the rails by track feeders. How to meet that non-economic requirement in the most economical way will be discussed below.

As regards the insulated or trolley-wire side, the calculations of the most economic sizes of feeders should follow general principles. It has been assumed that the substation positions have been settled from a load plan of the route; the values of load factor and form factor for each section of the route fed from each substation will be available from the same data. The trolley wires are usually run in half-mile sections fed through feeder pillars at that spacing and electrically in parallel with the feeder cable. On lightly loaded terminal lengths the trolley wires may afford sufficient conductivity to dispense with a feeder cable. The section pillars then have only links and fuses connecting the successive trolley wire sections.

The length of route to be fed from one substation may generally be taken as half-way to the next substation on each side. Usually it will be near enough to assume that the whole load of all the cars on the route between two substations is concentrated at the centre of the length and equally divided between the two substations. Allowing for the parallel trolley wires the appropriate economic (or nearest standard) size of feeder cable can be selected. As the feeder will be tapped at each half-mile, theoretically it should be tapered down in section at each tapping point. Practically this may not be worth while. It is important to take account of the gradients . on the tramway as they determine the load imposed by each car. With any given service it can be assumed that the cars will be distributed over the route at about equal distances corresponding to their time headway and speed. Taking into account the stopping places and the gradients, the loading for each half-mile section can be worked out. The current taken is a maximum when starting, or when climbing a gradient, and a minimum when running "free" on a straight level or downhill road. The values of the currents can be obtained from the characteristics of the motor equipment for the known weight of car. A 1 per cent rising gradient adds 22·4 lb. per ton to the tractive effort required. Tractive effort, current, and speed are all inter-related. It is therefore possible to make

a time-current diagram for each half-mile section of each track for each of the intended services; thus a load diagram for typical days and for a whole year can be constructed; and if done for a fair number of representative sections of route it forms a proper basis for the general value of economic current density. But on particular sections, whether on account of gradients or of probable unusually heavy traffic at some times, it is necessary to determine the voltage drop in the trolley feeders. Motor characteristics are stated for a definite voltage at the motor terminals; usually 500 volts for tramway motors. If the voltage is lower, the current for a given tractive effort will not be altered, but the speed will be proportionately lower. Hence to maintain a designed service speed, the voltage drop must be limited. Besides rising gradients, the starting places have to be reckoned with. On single lines with turn-outs, there will be two cars starting simultaneously at every other turn-out under maximum service conditions; the double starting current should be allowed for in the limitation of voltage drop. (It may be noticed that on a single line with turn-outs the maximum number of cars—or car-trailer units— is equal to the number of turn-outs.) As regards gradients these are much heavier than in railway practice; 1 in 11 is not uncommon for considerable lengths, steeper ones up to 1 in 6 are to be found on British tramways. If regenerative control is used, a simultaneously descending car will supply some energy to the ascending car and so diminish the load on a feeder serving both. Even so it is advisable to reckon on the absence of a descending car. Without regenerative control there is no compensation for the extra current and energy taken by the climbing cars; roughly the extra energy is dissipated in braking the car on its descending journey. There is no doubt that regenerative control can effect considerable economy in energy consumption on hilly routes. Early difficulties in apply- ing it—forty years plus ago—seem to have been overcome. Whether the extra capital cost of the car equipment will be out-weighed by the saving in energy consumption is a matter for consideration in each case. Apparently the mercury rectifier is unable to return "regenerated" energy to the a.c. side, but this disability should not be insuperable.

Tramway centres whence several routes radiate are likely to have heavy peak loads, which can be calculated from the proposed service tables. It is often advisable to feed such

centres and short lengths of the radiating lines separately; either by special feeders or substations.

The rail side of the system requires special consideration. As above stated British regulations limit the potential difference between any points of the rails to 7 volts. This practically means 7 volts from the earth plates and rail connections at the substations. There are also limitations on the current density in the rails intended to reduce stray currents to negligible amounts for the protection of underground pipes and the like. Tramway rails have a cross-sectional area of 1 in.^{2} for each 10 lb. of weight per yd., and the resistivity of rail steel is of the order of 11 times that of copper, so that a rail of 100 lb. per yd. has a rather higher resistance than a copper conductor of 1 in.^{2} section. The two rails of a track are nearly equal to 2 in.^{2} section of copper. A mile of single track of 100 lb. rails has a resistance of the order of 0·025 ohm if the bonding is perfect. That cannot be reckoned as a permanent condition; it is safer to reckon the resistance as about 0·03 ohm per mile. If heavier or lighter rails are used, the resistance will vary accordingly. In order to allow for unusually heavy loading and other variations, it is as well to calculate for only 5 volts rail drop. Taking the track resistance at 0·03 ohm per mile, it is easily seen that the limiting product of amperes and miles for 5 volts drop is $5/0·03 = 167$ ampere-miles. Then plotting the car positions and their loads in amperes along a length of track fed from one substation, and adding the individual products of current by lengths, if this sum exceeds 167 ampere-miles the drop will be in excess of that prescribed, and some means of relieving the rails of part of the return current has to be adopted. This may be done by paralleling the rails by a copper cable. To shunt rails equal to, say, 2 in.^{2} of copper by a copper conductor is generally too expensive. It is much more economical to use negative boosters in each track feeder. The positions for the feeder connection to the rails can be deduced from the ampere-mile plot of the line, thus—

Starting from the substation, sum the ampere-miles until the limit is reached (167 ampere-miles in the cases supposed). Start again from that point as zero until the limit is again reached. That is the point for a track feeder which must be of sufficient section to take all the current beyond the first limiting point plus that from the track beyond to the next limiting point. The economical section of copper has

then to be chosen; from its resistance and the load, the characteristic of the negative booster which will automatically keep the rail potential within the desired limits is calculated. If the track is double, the feeder must take the required current from both tracks. As the cable has to be insulated for only a few volts to earth, a relatively cheap type is indicated; as the losses in it are generated by the booster the unit cost should be reckoned to include the booster capital charges and running costs; these two factors operate in opposite directions in the determination of the most economical section to use. Unboosted track feeders may be economical for short connections to heavy traffic centres.

The magnitude of this question of track feeding will be lessened by the use of rectifier substations which can be spaced at much shorter intervals along a line than was economical with rotary converters, or with stations generating directly at the tramway voltage.

The whole question of calculating both trolley and track feeders, as well as some other matters of economical distribution on tramways, was set out by the author in a paper published in the *Journal of the Institution of Electrical Engineers*,* in 1900. A good many details such as the prices assumed for cables, costs of generation, etc., are now out of date; but the method remains correct. Another very useful paper on the subject contributed by Messrs. J. G. and R. G. Cunliffe, is to be found in the same journal.† It will be obvious that the most economical spacing of rectifier substations should be considered in connection with the cost of feeders as affected by that spacing. More and smaller substations at shorter distances will cost more than fewer and larger ones at longer distances, but the feeder costs will vary in the opposite way, whilst there will be a saving in the energy losses in the l.t. feeders by abbreviating the feeders, i.e. by shorter spacing of the substations. As always, l.t. losses have a higher unit cost than h.t. losses.

Trolley-Bus Lines. The same general rules apply to these as above set out for tramways; with the important difference that there are two trolley wires, positive and negative, for each track, and no earthed rail return. Hence no questions of track feeders or rail drop arise. On the other hand both trolley wires have to be fed; and for similar loads and current densities the voltage drop on the feeders will be twice that of the single-pole

* Vol. 29, p. 692.   † Vol. 50, p. 704.

tramway system, and may have to be considered where it could be neglected for a tramway. This indicates that a closer spacing of substations may be economical for the trolley bus. The mercury rectifier is eminently suitable for the work.

The plotting of loads along a line from the intended service and consequent distribution of buses along the line, the motor characteristics, vehicle weights and speeds, is just the same in principle as for tramways. The rubber tyres of the bus admit of a greater ratio of tractive effort to vehicle weight than the steel tyres of tramway cars; hence the starting currents per ton may be larger. This should be taken into account in estimating the feeder loading. The voltage drop, especially on gradients, may not be negligible in respect to the required service speed; hence it may be found necessary to adopt a lower current density in the feeders than that calculated as most economical. In extreme cases it may be worth while to put boosters into the feeders in order to save cable costs. Automatic voltage regulation by the load is an alternative, if found to be economically practicable.

It seems possible to arrange the distribution as a three-wire system, one of each pair of trolley wires—the two of the "up" and "down" directions being connected in parallel—forming the mid-wire, the other two wires being positive and negative respectively on a double voltage distribution. The mid-wire would be earthed at the substations. Whether this has been tried or previously suggested, the author does not know. It would halve the expenditure on feeders. The obvious objection is that balancing between the two sides, i.e. between the "up" and "down" lines, might present difficulties; it seems worthy of some expenditure of ingenuity to realize so large a saving. One incidental advantage would be that the metal work of the bus could be connected to the earthed neutral wire, thus eliminating the (very small) risk of shock to passengers entering and leaving the bus which is insulated from earth by its rubber tyres.

Electric Railways. An embarrassing number of supply systems is in use on electric railways; d.c., single-phase, and three-phase a.c. at several voltages for each kind. The choice between them is usually determined by other considerations than that of distribution economy. From that consideration alone, there can be little doubt that the single-phase high-tension plan provides the most economical method where overhead trolley wires are admissible. On the very extensive system of

the Pennsylvania Railroad, 11 kV. single-phase at 25 periods has been adopted after successful use on the New York and Newhaven line for many years. On the Pennsylvania Railroad the main feeders are overhead lines, with a few sections of cable, at 132 kV. One of the factors against the single-phase system is that for the sake of the motors the frequency should not exceed 25 cyc.: on some Continental lines 16·66 cyc. is used. This involves the expense of frequency conversion if the general supply at the standard 50 cyc. (60 in the U.S.A.) is to be used; alternatively, special generators must be installed.

In this country the single-phase system had a prolonged and successful run on some of the suburban lines of the L.B. & S.C. Railway. It has been supplanted there by a d.c. third rail system since the grouping of the railways.

The Committees which have considered and reported on the question of standardizing railway equipment in this country have recommended d.c. working. The deciding factors, as they appear to the author (who admits that his views may not be generally accepted), are: first, that the overhead single-phase wires produce considerable disturbance in adjacent communication circuits and that the cost of reconstruction of such circuits for the avoidance of disturbance would be prohibitive; and secondly, that tunnels, over-bridges, and some station structures give insufficient clearance for the overhead wires; again the reconstruction to suit would be impracticably costly. However that may be, it must be accepted that a.c. systems are barred in Great Britain.

Elsewhere such conditions may not operate. The single-phase h.t. system is probably the only one economically possible for long lines with light traffic, such as exist or may be built in Australia, South Africa, and similar countries. A notable example is the long line which serves the iron-ore mines in the far north of Sweden. There the abundance of water power has been a deciding factor in the adoption of electric traction; but h.t. transmission and distribution were essential to economical utilization of the power. Rotary substations and l.t. distribution would have been too expensive.

It is true that single-phase traction motors are more expensive and somewhat less efficient than equivalent d.c. motors, but that disadvantage is outweighed by the superior economy pertaining to a.c. high-tension distribution over long distances.

In the design of the line equipment of single-phase railways the voltage drop on the rails has to be considered. Steel rails have a high impedance due to their magnetic permeability and to the "skin effect." That has to be compensated by track feeding through transformer boosters. Undue voltage drop causes a sensible straying of the return current through the earth, which may interfere with communication circuits especially with telegraph circuits using earth returns. This is distinct from the inductive effects from the overhead wires. Both kinds of interference were successfully countered on the suburban lines of the L.B. & S.C.R.; there is no reason to consider it as a serious matter in general. The higher the trolley wire voltage, the smaller will be the currents pertaining to a given kilovolt-ampere train loading; therefore the smaller the effect of the rail impedance : on the other hand, the electrostatic induction on open wire communication circuits increases with the trolley voltage. There is also electromagnetic induction upon open wire circuits from the loop formed by the trolley wire and the rails. This is reduced by increasing the voltage, just as is the rail drop, because both are current effects. The subject is a complicated one which has occasioned mathematical discussions which few can follow. Experience shows that the interference can be reduced to negligible dimensions by co-ordination between the parties interested.

Three-phase current is in use on some Italian railways (perhaps elsewhere). Whilst it is a practical success, the disadvantages of a double overhead conductor, and some complications of the control and motor equipment make its extended application improbable.

There have been some trials of various methods of converting a.c. to d.c. on locomotives, in order to combine economical distribution and high motor efficiency. None seems to have survived or achieved extension.

In this country direct current at several standard voltages has been recommended by the Committees which have studied the subject at the behest of the Government. The voltages have been chosen to make inter-running possible between lines using different voltages; for example, 1 500 and 3 000 volts, the motors being run in parallel on the low and in series on the high voltage.

The most extensive electric railways in this country are those of the London Passenger Transport Board, and of the

Southern Railway. Both of these use d.c. at about 600 volts supplied to the trains through conductor rails. The L.P.T.B. lines are mostly underground and tube railways. They have two conductor rails, positive and negative. The special reasons for adopting two conductor rails, were: (*a*) to leave the running rails free for track circuiting; and (*b*) to avoid the restrictions on voltage range on the track rails prescribed by the pertinent regulations. The pioneer London tube railway, the City and South London, had only a positive conductor rail, the return was by the track rails. It might have been expected that the iron tube would have prevented any notable amount of stray current. That was not the case, and measures had to be taken to eliminate risks to gas pipes, water pipes, and of interference with telegraph circuits. The other reason—leaving the track rails free for track circuiting—arose because track circuiting at the time was d.c. only. To-day a.c. track circuiting is practicable and is used on the Southern Railway and other lines. The rails on consecutive block sections are isolated for the signalling current by impedance bonds which allow the d.c. traction current to pass. It is unlikely that double conductor rails will be used in future electrifications. It may be remarked that the very extensive negative rail network of the London Underground lines gave rise to some undesirable and unexpected short-circuit phenomena. It is probable that the network has since been cut up into smaller sections, but the author has no certain knowledge of this.

As with tramways and trolley buses, the mercury rectifier has supplanted rotary converters on the most recent railway electrification work. A notable example is that of the Southern Railway extensions to Brighton and other South Coast towns. The facility of feeding rectifier substations from transmission lines at standard frequency—such as those of the C.E.B. grid— makes it certain that most future railway electrifications in this country will use rectifiers with remote control from few centres as is done on the Southern Railway. Rotary converters for railway use have heretofore been generally worked from 25 cyc. sources, involving special generators or frequency conversion, both rather expensive. The erection of new generating stations for railway purposes only is improbable in Great Britain.

For the calculation of the distributing equipment on a railway, the same general method as outlined for tramways should be adopted. The working time-table of the proposed services,

with car and train weights and speeds, gradients and station positions, in conjunction with the motor characteristic curves, permit of a time load curve being drawn for each section of the line. The load spacing is more definite on a railway, as each block section can have only one train unit in it at any one time. But the load of such a car or train may vary between very wide limits. For example, a single motor-equipped car may weigh about 60 tons; units of one motor car and two trailers, and multiples of such units, may run up to 300 or 400 tons; whilst freight trains drawn by locomotives may run up to 1 000 tons. It has to be observed, however, that multiplying the number and reducing the weight of trains is definitely advantageous to economy in electric operation, whereas with steam traction it is advantageous to run fewer and heavier trains. The service and working time-tables for an electrified railway should not be a copy of the steam working schedules. From the data named above, a 24-hour load diagram can be prepared for each block section on each length of route over which the service is identical, generally for each length between junctions. At stations, allowance must be made for the starting currents of trains which call at each. Allowances may have to be made for slowing and accelerations at curves and junctions. Evidently where there are several block sections between two stations the whole station-to-station length can be treated as one loading section. A preliminary step is to make a speed time curve for each class and weight of train on each start-to-stop run shown by the working time-table. This has then to be translated into a load/time curve from the motor equipment characteristics of each class and weight of train on each section. From these two sets of curves a load/time diagram of the whole line can be constructed, showing the load distribution along the line at a number of stated times as well as the total load. It will probably be necessary to make these for several typical days; special attention being given to the seasons of maximum traffic density. The annual load and form factors can then be worked out for each section of the line; and the total load and annual consumption for the whole line.

The desirable positions of substations can be readily seen from such an assembled time/load curve. Junctions whence two or more lines radiate are evidently suitable for some of them. Some positions may be more or less fixed by the run of the available h.t. transmission lines; where these are unfavourable

feeder lines at, say, 33 kV. may be run along the railway to better sites. In some cases it may be advisable to run such a h.t. line along most of the railway, when the substations may be sited without regard to the main transmission system.

An important, may be a deciding, factor in the spacing of the substations is the voltage drop in the conductor and running rails. The running rails are usually no heavier than tramway rails; they may be lighter. The train currents will usually be heavier than those encountered on tramways; their magnitude depends upon the voltage adopted. The sectional load diagram will indicate what maximum current has to be allowed for on each section fed from one substation. The limiting ampere-mile product for the known rail resistance can then be applied to each length between substations in order to see whether and where feeders are necessary; just as explained in the case of tramways. Where track-circuiting is to be used, the feed to the two running rails of each track has to be at the centre point of an impedance bond between them; at the ends of block sections impedance bonds must also be used. It may or may not be permissible to bond together the running rails of both tracks on double lines. This matter of impedance bonding, the impedance required in each position, etc., must be settled in conjunction with the signalling designers. The resistance of the impedance bonds has to be taken into account when reckoning the ampere-miles for the drop in the return, as an addition to the rail resistance. It will be appreciated that feeders and boosters to deal with heavy currents are expensive; so that the siting of substations at such distances as to minimize track feeding may be well worth while. Where unavoidable, the most economical current densities—i.e. cable sizes—will be calculable on the same lines as for other cables. There are no regulations limiting the voltage drop on the track rails of main line railways in this country. As indicated above track circuiting imposes limitations.

Where the line equipment is third-rail, the rail steel should be of higher specific conductivity than that of track rails; instead of one-eleventh of that of copper it may well be only one-sixth or one-seventh. Hence conductor rails of the usual 80 to 100 lb. per yard are equivalent to copper of from 1·3 in.^{2} to 1·66 in.^{2}. The practical limitation of voltage drop is that the voltage delivered to the heaviest train at the greatest distance from a substation must be sufficient to maintain the

scheduled speed of that train; the current required for that speed and load can be determined from the motor equipment characteristic curves. As it will obviously require large copper sections of feeder cables in parallel with the conductor rail to materially reduce the voltage drop in them, it will rarely be economical to take conductor rail feeders beyond the nearest point of the sections fed from each substation. The substation voltage regulation should preferably be designed to take care of the voltage drop to the trains. Voltage variation of railway supply may be considerably greater than that permissible on general distributing systems. The conductor rail is a wearing part; it loses metal by corrosion as well as by attrition; data on both kinds of loss are available and should be used to predict the conditions over a reasonable term of years. From considerations of distribution economy, one would prefer the heaviest rail compatible with the mechanical track structure; and would take steps to secure the most efficient form of bonding conductor rails. The limit of permissible loss by wear is a rather small fraction of the whole section of the rail. Supposing that the allowable wear is defined by some minimum section—which should be that at which the resistance and voltage drop just verges on the maximum allowable—the actual section selected should be that minimum plus the corresponding loss. For example, if the limiting section is that of an 80 lb. rail, the rail chosen might be of 100 lb. per yd., when the useful life of the rail would be that in which the wear amounted to 20 lb. per yd. If one could foretell the rate of wear it would be possible to state an equation between the cost of the allowance for wear and the recurrent cost of renewing the rail: evidently the larger the allowance made the longer would be the intervals between renewing the rails. Such a calculation should take into account the scrap value of the old rails. The cost of renewals is clearly a working expense which should logically be apportioned to the traffic over the useful life of the rail.

Summing up: after the siting of substations there is little scope for distribution economy on the positive or conductor rail side; but there may be some on the track rail or negative side, occasioned by the need to comply with track circuiting limitations. The working voltage is the main factor. Possibly 1 500 volts may be permitted on new third-rail lines; which puts distribution economies into a very different proportion from those prevailing on a 600 volt line. It may be remembered

that 1 200 and 1 500 volts have both been used in this country for some time. Insulation and guarding involve greater expense than with 600 volts, which must be considered when comparisons are made of the economical merits of different voltages. Permanently good bonding of both conductor and running rails is very desirable : permanency is more difficult to ensure in the running rails. A standard commonly adopted is that the bond resistance should not exceed that of 6 ft. of the rail. That will add 10 to 15 per cent to the resistance of the rails ; new bonding should be much better. Welded joints— the best electrically—are not in favour with railway men ; there is less reason to object to them in conductor than in running rails. Available experience will be the best guide to the method to be adopted.

RAILWAYS WITH OVERHEAD CONDUCTORS. For 3 000 volts and more, overhead wires must be used. The higher voltage permits of longer intervals between substations without introducing the complications of rail drop in excess of that allowed. The overhead conductor will have a relatively small equivalent copper section compared with that of a third rail. The most economical overhead construction in a given case needs careful consideration. For railway speeds it is necessary that the contact wire shall be as nearly as practicable at a uniform level from the rails. The sag and rise tolerable at tramway speeds are not so at railway speeds. Hence railway overhead work is designed on the catenary principle ; the contact wire carried by means of dropper wires from a supporting or catenary wire which takes all the sag. The contact wire is a wearing part ; it has to be renewed at intervals. There is therefore some size which in the long run will prove less expensive than either a larger or smaller size. Evidently the larger the new wire, the longer will be its life, but the proportion of copper scrapped at each renewal will be greater—other things being equal—than with a smaller wire. With the smaller wire the first cost will be less, the amount of scrap less, and the renewals more frequent. The calculation for the most economical size involves the question of rates of wear, the labour cost of renewals, etc., hardly capable of solution excepting from experience under similar conditions. As the catenary wire is essential, it should be made the main conductor ; the most economical size of the contact wire then involves no material element of distribution losses. The most economical current density in the catenary

can be calculated exactly on the same principles as for a cable. That will give differing sections for different lengths of the line, varying with the mean current loading as well as with the load and form factors. As the catenary may be composed of one or two stranded cables, it will be found that the price per ton of copper keeps more nearly constant over a range of sizes than is the case with insulated cables. The calculated current density gives directly the rate of voltage drop per mile at any load; therefore it can be seen, without calculating out the sizes, on how many lengths of the railway that density at the maximum load for the length will produce a voltage drop within the permissible limits. That supposes that the substation sites have been previously fixed. But the calculation may be used as an element in settling the best distances between substations. Theoretically, if a given distance and the most economical current density gives too high a drop *at top load* midway between two substations, they should be closer together. This may mean more and smaller substations, with some increase in their cost. By trial and error with a few variations one can soon arrive at the best combination. Where the traffic is densest, the substations should be closest.

It will be seen that the rule of "most economical current density" prescribes different catenary sizes on differently loaded lengths of line. Within limits there is no objection to such a variation. But at certain steps in size the strength and cost of the supporting structures have to be altered. The increase of cost at a given step-up may quite well be reckoned in with the cost of copper per ton; so that the most economical current densities will also go by steps. One would naturally begin with the lightest catenary anywhere sufficient, take out the cost of the necessary supporting structures, and regard that as a standard minimum. Then any larger catenary will need a stronger supporting structure, which will suffice up to another size of catenary. So one can get out a few standards suiting different situations, each standard structure being suitable for a certain range of catenary sizes. Catenary wire may be of steel and aluminium strand, an alternative to copper worthy of consideration.

It is unnecessary here to enlarge on certain features of the work which do not affect the economy of distribution; such as the special work needed at junctions, cross-overs, sharp curves, and so on; the cost of these will vary somewhat with the catenary size selected, but not very much.

In all traction systems, tramway, trolley-bus, and railways, a question arises as to whether consecutive lengths of trolley-wire or conductor rails should be connected across at feeder pillars or substations, as the case may be. There is a manifest advantage in connecting them, as there will be some savings in voltage drop and losses. On the other hand a fault on one length may cripple all the lengths with which it is in parallel. Such effects can be minimized or prevented by making the connections through suitable fuses or circuit-breakers. It is quite useful to be able to run a length of line with some of the substations cut out during light load hours; or whilst cleaning, etc., are in progress. On railways it certainly seems right to connect up the whole length of conductor between two neighbouring substations. The track-circuiting may determine whether this should be done; consecutive block sections must have impedance bonds between their ends; and there is some limit to the traction current which such bonds can carry safely, also some limit to the paralleling of impedance bonds as regards the a.c. conductance which can be permitted from rail to rail.

As an example of current practice, the distributing arrangements on the newly electrified lines of the Southern Railway are of interest.

The information has been very kindly supplied by Mr. Alfred Raworth, the Electrical Engineer for New Works to that railway.

The mercury arc rectifier and a general standardized spacing and lay-out have been adopted.

The main transmission line at 33 kV. runs along the railway; it is fed from C.E.B. substations and other sources as convenient; there may be alternative feeds at each receiving centre.

On double track lines the standard arrangement is shown in Fig. 7.

Substations are 3·5 miles apart. Conductor rails are of 100 lb. per yd. section, of steel of about one-sixth the conductivity of copper, the resistance per mile, bonded, being 0·03 ohm. The "up" and "down" line conductors are fed in parallel through circuit-breakers at the two substations; divided at the mid-point, and there paralleled through four circuit breakers and a tie-bar. Neighbouring sections are fed through circuit-breakers from the substation bus-bars, so that

all the conductor rails fed by a line of substations are in parallel.
Thus on each length of 3·5 miles there are four sections of
conductor rail, two on each track, normally all in parallel.
The two running rails of each track have a resistance of 0·03
ohm per mile. They are of lower conductivity steel. The four
rails are connected in parallel through impedance bonds at
the substations, and similarly bonded across at several points.
The extent of this cross-bonding is limited by the requirements
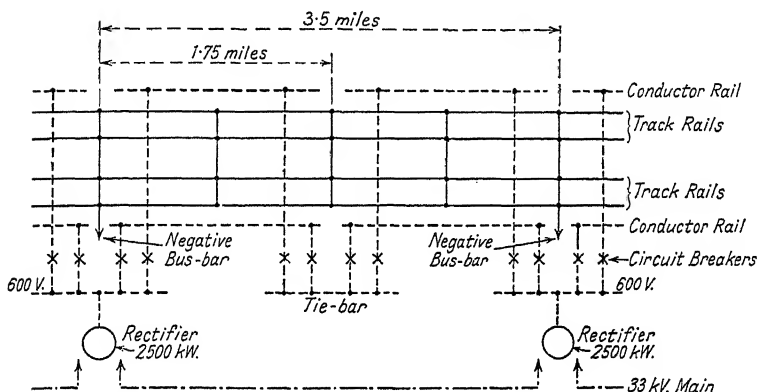
Fig. 7. Southern Railway Electrified Main Line—
Standard Section

of track-circuiting; the rail-to-rail resistance to the a.c. signal-
ling current must be sensibly higher than the resistance through
the wheels and axles of a single vehicle. This limitation also
affects the extent to which the track rails of contiguous sections
can be bonded into electrically continuous lengths. There are
no rail voltage drop regulations. The railway communication
circuits all have metallic returns.

It will be seen that a short circuit on any one of the four
lengths of conductor rail isolates that length by the tripping
of the circuit-breakers at the substation and at the midway
tie-bar. The maximum load of a 12-car train is about 4 400
amperes; the substation breakers are usually set at 6 000
amperes and those at the midway tie-bars rather lower. Where
there is a cross-over road, special arrangements of conductor
rails and switches permit of by-passing an obstructed section
by single line working.

The maximum drop of voltage occurs when two 12-car trains

are passing at the mid-point of a section; it is then about 13 per cent, a condition lasting only a few seconds. The substation pressure is 660 volts, and the train motor designed pressure is 600 volts; hence the standard lay-out imposes no voltage drop appreciably detrimental to the train speeds.

The substation equipments each consist of a mercury arc rectifier rated at 2 500 kW. d.c. output, with the necessary transformers and switchgear. They are remotely controlled from a few central points. No spare plant is provided, as the substations are in parallel on both the h.t. feed and the l.t. rails. They share the loads; any one of them can be cut out without interruption of supply; some are regularly cut out during light traffic periods, as well as for inspection and overall. There will eventually be 120 of these standard rectifier substations in operation when the electrification schemes now authorized are completed.

The capital and running costs are considerably lower than those of rotary converter substations; routine attendance is only for periodical inspection and the like.

Measurements over yearly periods of the substations now in service show an overall efficiency approaching 90 per cent from the feed to the h.t. lines to the substation d.c. output. There are no meters on the trains, but it is evident from the data given that the losses on the rail side of the substations may be of the order of 3 per cent to 5 per cent, a result achieved it may be noted, without using any auxiliary copper feeders to either the conductor or the track rails.

No troubles due to electrolytic corrosion have been experienced. Although the rail drop and current density are both much higher than those prescribed for tramways, this freedom from troubles may probably be credited to the fact that the running rails are on creosoted sleepers, and that the rail-to-rail resistance of each track has to be kept up by the requirements of track circuiting. Tramway rails, buried and seated on moist concrete, naturally make much better earth; also they are near neighbours of numerous buried pipes, etc.

Regenerative Control and Braking. This is another feature common to traction systems in general, but presenting different aspects in respect to railways on the one hand, and road systems on the other. With regenerative control, part of the energy of a car or train which is descending a gradient, or is being decelerated, is returned to the line and used either to assist

cars or trains which are ascending a gradient, or accelerating. In both cases the motors acting as generators are in effect brakes; but the two conditions of descending a gradient and braking for stopping are of different magnitude and duration. Obviously a means of braking which results in the return of some energy to the line instead of dissipating it in grinding brake block, wheels, or drums, thus wasting the kinetic or gravitational energy in heat and wear, is economical. The most pronounced case for regenerative control arises on hilly routes, especially where the gradients are steep and long. If one car or train is descending such a gradient and another one is ascending it, the work done by gravity on the descending car may be in part transferred to the ascending car, so reducing its call upon the source of supply. The action is analogous to that on a line worked by cable linking the descending and ascending vehicles. In the electric case the regained energy may be utilized on another section of the system, or returned to the supply mains, if there is no call for it from other vehicles.

Gradients of a degree making this mutual assistance scheme worth while are more common on tramway and trolley-bus routes than on railways; but there are some railways on which the saving has been one of the factors in determining electrification.

Regeneration when braking for stops may be worth while on tramway and trolley-bus routes, where stops are several per mile, and it is pretty certain that there will be a car or bus starting near by. But it has not appealed to traction engineers for that purpose; one reason being that regenerative braking can hold a car to a safe speed on a falling gradient, but cannot stop it. So a mechanical brake of some kind must be provided.

On switchback routes there is a real saving possible. It has been recently stated that on a hilly system of trolley bus routes in an English town the saving is of the order of 20 per cent of the energy used. Regenerative control is somewhat costly. The electrical equipment is more complicated; and motors have to be of greater capacity, because going downhill they are working as generators, instead of having a chance to cool down. The maintenance costs are increased somewhat; so it becomes a question of balancing the extra capital and working costs against the value of the energy and brake-gear wear saved, and, perhaps, safer operation.

The introduction of the mercury rectifier tends against regeneration, because this apparatus is incapable of returning

regenerated energy to the supply mains. This difficulty may be got over by cross connecting the l.t. sections served by a rectifier substation, if the lay-out is such that descending vehicles on one side may help ascending cars on the other. That is a reason for inter-connecting sections, where such conditions obtain. There seems to be a possibility of using "grid control" in the rectifier which will make it able to return regenerated energy. No difficulty arises where the supply is through rotary converters.

There is a considerable literature on this subject of regeneration which should be consulted by anyone interested in the matter.

It is hardly necessary to point out that the adoption of regenerative control may have an important bearing on a distribution lay-out for maximum economy.

Temporary Supplies. In most cases these are only "special cases" because the load occurs for short periods. Show yards, exhibition buildings, sports grounds and the like, may call for large loads for a few days per annum. Frequently such places are in "thin" areas, so that the permanent mains, etc., are inadequate for the temporary load. One way of providing for such demands is to have one or more portable substations on lorries or trailers, with transformers and switchgear. Then the only permanent provision necessary is a high-tension main terminated in a kiosk or street-box adjacent to the place. Such portable substations are useful for other purposes, for example for taking the place of a regular transforming substation during overhaul, changing of gear, and the like; for giving an emergency supply to a factory where the engine or private plant has broken down; or for occasional flood-lighting where the permanent distributors are insufficient for the temporary load.

For seasonal illuminations, such as, e.g. those common along sea-side promenades, the main consideration is to employ material easily erected and removed, and able to stand exposure and handling several times before becoming unfit for use. As the life of such material is likely to be short, low first cost is a consideration. A series-parallel arrangement of the lamps, i.e. groups of lamps in parallel, successive groups in series, fed at high-tension, avoids the use of large cables and the loading of the permanent l.t. mains; besides affording facilities for independent switching and ensuring equality of voltage between the groups however long the string of festoons may be.

CHAPTER XV

LOAD, LOSS, FORM, AND DIVERSITY FACTORS

THE load factor of a distributing system, or of any portion thereof, in terms of the maximum load (kW.$_{max}$), total energy delivered (kWh.$_{total}$), and length of the period under consideration in hours is given by equation—

$$\frac{\text{kWh.}_{total}}{\text{kW.}_{max} \times \text{hours}} = \text{Load factor.}$$

It is usually expressed as a percentage; for example, an annual load factor is

$$\frac{\text{kWh.}_{total} \times 100}{\text{kW.}_{max} \times 8\ 760} = \text{Load factor per cent,}$$

and for any other period the multiplier of the maximum load in the denominator is the number of hours in the period, 168 for a week, 24 for a day, etc.

Another way of expressing the load factor is

$$\frac{\text{kWh.}_{total}}{\text{kW.}_{n}} = \text{Equivalent hours use of the maximum load.}$$

This may be called a utilization factor, the number of hours in the period being understood.

For estimates of copper losses the loads must be measured in amperes, which automatically takes into account power factor and voltage fluctuations. The load factor calculated from amperes and ampere-hours may not be identical with that calculated from kilowatts and kilowatt-hours unless the measurements are taken at a point where the voltage and power factor are constant.

Knowledge of the maximum current and load factor are not sufficient to assess the copper losses in mains and transformers, because those losses at any time are proportional to the square of the current; consequently the losses vary with different shapes of load diagrams which have identical load factors. But the load factor sets an upper limit to the ratio of the losses to those pertaining to the average current maintained constant over the whole period in the same conductor.

That maximum occurs when the load factor is also the time-utilization factor; i.e. if the load factor means that the maximum current was constant for the fraction of the time denoted by the load factor, and no current passed during the remainder of the time : in other words, if the load diagram is a rectangular one based on the fraction of the time. For example, if the load is steady for 6 hr. per day and zero for the other 18 hr., the load factor is 25 per cent. The average current over the whole day is one-quarter of the maximum current. For a given conductor of resistance R ohms, calling the average current I amperes, the losses in watt hours over the 6 hr. are—

$$(4I)^2 R \times 6 = 96I^2R.$$

The losses in watt hours with the average current over the whole day would be
$$I^2R \times 24.$$

i.e. the losses are four times as great for the 25 per cent load factor of a rectangular load diagram as for the average current delivering the same number of ampere hours at 100 per cent load factor.

Similarly, for a 50 per cent load factor given by a load constant for 12 hr. per day and zero for the other 12 hr. The ratio of losses to those pertaining to the equivalent average load is 2. These ratios 4 and 2 respectively are the reciprocals of the load factors : they are also the mean squares of the loads taken over the whole period and are the maximum values of the loss ratios for load diagrams giving load factors of 25 per cent and 50 per cent respectively. Any other load diagrams having load factors of 25 per cent or 50 per cent will have lower loss ratios. In every case and for every load factor the loss ratio is the mean square of all the current values taken over the whole time.

It was shown in Chapter III that the most economical current density in a given conductor is that which makes the value of the losses over a given period equal to the fixed (capital, etc.) charges which are directly proportional to the cross-section (or weight per unit length) of the conductor; and that such a density is readily calculable for a steady current, i.e. for 100 per cent load factor. The question arises, what current density referred to the average current will keep the copper losses equal in value to the fixed charges for any

other load factor? As just shown the last words should be "any other load diagram."

It has been shown that for a 25 per cent load factor rectangular load diagram the losses are four times as great as those with the equivalent average load. If the conductor is doubled in section, the losses will be halved, i.e. they will be twice as great as those for the average current in the original conductor.

Also the capital, etc., charges on the conductor will be doubled; hence the required condition of equality of value of losses and fixed charges is reached. A similar calculation for the 50 per cent rectangular load will show that the divisor for the original 100 per cent load factor current density is 1·414, i.e. the square root of the mean square pertaining to the load diagram. These are illustrations of the rule that the maximum divisor of current density (or multiplier of copper section) found for 100 per cent load factor is the square root of the reciprocal of the load factor. The minimum value for any load factor approaches unity, but only reaches that value for 100 per cent load factor. For example, if a load is constant over all but a few minutes of a day, but during those few minutes reaches four times the average, giving a load factor of 25 per cent, the losses over the day will be very little greater than if the average had been maintained at a steady value over the whole day.

Loss Factor. The factor required to answer the question put above is the square root of the mean square of the ordinates of the load diagram. For any load diagram the mean square of the current ordinates is the multiplier of the losses due to the average current in the same conductor. It may therefore be called the *Loss factor* pertaining to that shape of load diagram. The square root of the mean square is the divisor of the 100 per cent load factor current density for maximum economy to be applied to the average current in order to have maximum economy with the load diagram under consideration. That is usually known as the *r.m.s.* or *form factor* pertaining to the load diagram. Both factors are geometrical properties of the particular shape of load diagram in relation to the length of its time base; i.e. the whole period of the load cycle. Once found for any shape of load diagram they can be applied to any other of the same shape, whatever the scales.

The mean square of a diagram is obtainable by summing the products of each load squared and the time each lasts, and

dividing the sum by the whole time. The process of squaring and summing is tedious, especially if a large number of diagrams have to be dealt with. The work can be shortened by plotting the loads in order of magnitude, giving each a time dimension equal to the total number of hours during which that load was experienced over the whole period. Then each load value has to be squared only once. It may happen that the outline of the stepped diagram approximates to a curve the equation to which is known, so that the mean square can be found without much trouble.

A more radical labour saving device is to plot the load diagram to polar co-ordinates which does the squaring and multiplying automatically. The basis of this method is the fact that the area of a sector of a circle of radius r and included angle θ is $r^2\theta$, θ being measured in radians. Hence the area of a load diagram plotted to polar co-ordinates is proportional to the sum of the products of the squares of the individual loads and the time duration of each.

The procedure is as follows—

Draw a semicircle with a radius which represents to a convenient scale the maximum load. Divide this semicircle into convenient fractions representing units of time, the whole arc of 180° representing the whole time covered by the load diagram. If for a day, one hour will be represented by an angle of $180°/24 = 7\frac{1}{2}°$.

Draw from the centre of the semicircle radii of lengths proportional to the loads on the same scale as that chosen for the maximum load, i.e. the radius of the semicircle. For example, if the first load was 100 and it lasted for $\frac{1}{2}$ hr., mark 100 on the base of the semicircle, draw another radius of the same length towards the division 3·75° or half an hour and the arc joining them. (It will be convenient to mark out the semicircle in hours and fractions corresponding to the time divisions of the original load diagram or log sheet.) If the next load entered is 150, and it lasted 1 hr., prolong the second radius to 150 draw an arc of $7\frac{1}{2}°$ at that radius, i.e. to $1\frac{1}{2}$ hr., and so on.

Having completed the polar diagram, measure its area by planimeter, and call this area A. Since the area of the whole semicircle to radius r (the maximum current) is $r^2\pi/2$; $2A/r^2\pi$ is the ratio of the actual copper losses to the losses which would be produced by the maximum load for the whole time. It is the load factor of the losses, a geometrical property of the

shape of the diagram, the area of which is a measure of the losses.

If the fraction—

$$\frac{\text{Area of polar diagram}}{\text{Area of semicircle}} = \frac{1}{n}$$

and the maximum current is I_{max} amperes, the mean square of the load diagram current is

$$I^2{}_{max}/n = \text{mean square; and the r.m.s. is } \sqrt{(I^2{}_{max}/n)}$$
$$= I_{max}/\sqrt{n}$$

that is, the current which maintained constant over the whole time would give the same I^2R losses as those pertaining to the actual load current.

It may be noticed that if a recording ammeter chart is circular, the pen movement is radial and directly proportional to the current, and the zero is in the centre of the chart; the area of the record gives the required data, a rather useful property of the disc chart.

TABLE IV

CUMULATIVE TIME LOAD FOR A TYPICAL DAY, DERIVED LOAD-FACTOR, R.M.S. LOSSES (AMPERES² × DURATIONS) AND LOSSES LOAD-FACTOR

Load Amperes	Duration Hours	Ampere-Hours	Amperes² × Hours
1 000	0·25	250	250 000
950	0·50	475	451 250
900	1·00	900	810 000
800	1·25	1 000	800 000
750	2·00	1 500	1 125 000
600	2·00	1 200	720 000
500	3·00	1 500	750 000
400	3·00	1 200	480 000
300	2·00	600	180 000
250	5·00	1 250	312 500
200	4·00	800	160 000
	24·00	10 675	6 038 750

Average load	445 A	
Load-factor	44·5%	
Mean square	$\dfrac{6\ 038\ 750}{24} = 251\ 615$	
R.m.s.	501·6	
Load-factor of (ohmic) losses . .	$\dfrac{6\ 038\ 750}{24\ 000\ 000} = 25\cdot16\%$	

Table IV is an example of a "Cumulative Time" load table for one day, which might represent an average day from a summation of the log readings of a number.

Fig. 8A is the corresponding load diagram, and Fig. 8B the same load diagram converted to polar time ordinates, the 24 hours being represented by a semicircle, 7·5° per hour.

Table IV shows the arithmetical derivation of load factor,

Total Output:..........10,675 Ah.
Maximum Load:.......1000 A.
Load Factor:.........$\frac{10,675}{24,000}$ = 44·5 %
Average Load:.....445 A.
R.M.S. of Load:.....502 A.
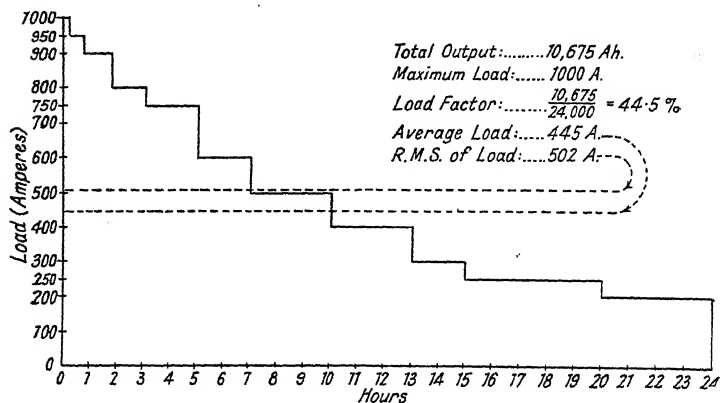
FIG. 8A. LOAD DIAGRAM FOR A DAY
Cumulative Time for each value of load

products of amperes² × hours, and load factor of the ohmic losses. The total of the amperes² × hours, and the consequent load factor of the losses, could be obtained by planimeter from Fig. 8B.

In this example the load factor is 44·5 per cent, the average load 445 amperes (maximum 1 000), the mean square of the load is 251 615, the r.m.s. 501·6, and the load factor of the losses, 25·16 per cent.

The load factor applies only to ohmic losses in a circuit of constant resistance. It would apply up to the first point at which the circuit branches, which might be the bus-bars of a station or substation, or the end of a feeder; and is for the phase on which the logged values were read.

The load factor of the losses is a useful datum as an element in the cost of the losses and the generating plant needed to provide them. Examples have been given in Chapter III.

In the foregoing discussion, "load" and "current" have been used rather as equivalent. Evidently for the purpose of

calculating copper losses, economical sections of copper, etc., the current in amperes is the quantity required.

For the purpose of evaluating energy losses, the product of the mean square value, the copper resistance, and the time in hours gives the losses in watt-hours, and this product, divided by 1 000 in kilowatt-hours.

The current-time diagrams for an untapped continuous

Area of complete half-circle represents 24,000,000 $A^2 \times$ hours.
Area of segments represents 6,038,750 $A^2 \times$ hours; hence the load factor of ohmic losses = 25·16 %.
Mean square = 251,615.   R.M.S. = 501·6.
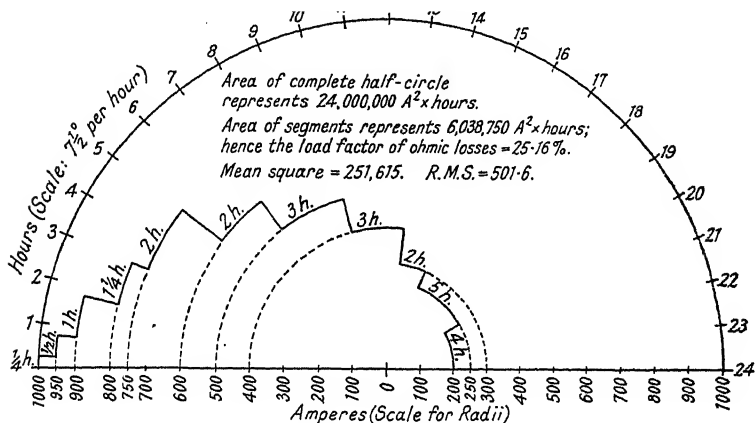
FIG. 8B. LOAD DIAGRAM TO POLAR TIME ORDINATES

current feeder free from leakage will be identical at both ends. With alternating current they may not be identical. In general the ampere-hours put into a feeder will be more than the ampere-hours delivered by it. Capacitance adds to the load a leading current in quadrature which is proportional to the voltage, i.e. is constant for constant voltage. Inductance adds a lagging current which is proportional to the load current. At a certain load the two effects may cancel out. In cables the capacitance predominates, in overhead lines the inductance. Neither effect is very important in short lines at moderate voltages, such as are found in most distribution systems. The charging or capacitance current may have other effects during light load periods when it is a sensible part of the whole current. These effects are outside the scope of this book.

Voltage regulators at the delivery ends of feeders impose some current load on the feeders which does not appear in the output beyond the regulators. As this load is usually a

maximum at the peak loads it makes the maximum loss rate higher and the load factor of the losses lower, part of the cost of distribution which may be of consequence. This should be taken into account when deciding whether to make shift with a feeder which needs a regulator to maintain voltage at peak loads, or to put in a larger feeder (or to supplement an existing one) which will need no regulator.

The simple lines of calculations for copper losses, most economical section, or current density, as set out here and in other chapters, are subject to modification in application to large cables carrying alternating currents on account of skin effect and proximity effect. Skin effect increases the resistance beyond that measured with continuous current. Proximity effect is more complex, as it includes inductive effects. The single-conductor cables which are now used for 66 kV. and higher pressures are also not amenable to simple calculation. The currents induced in the sheaths add to the load currents; how much depends upon the nature of the sheathing, the spacing of the cables, and the arrangements of the sheath bonding. The dielectric losses are not negligible and, as they increase with temperature, the safe loading of such cables is more definitely limited by heating than by other factors. Current literature will give some information, but experience is rather scanty as yet. Problems involving the use of such cables are usually presented in such form as: for the transmission of a load of M kVA. from point A to point B at a load factor of N per cent, what will be the most economical voltage type and size of cable?

This is a transmission rather than a distribution problem, but there are already several instances of such pressures being used in the first stage of distribution. One of the latest reported is in Chicago. A single-conductor oil-pressure cable line working at 66 kV. has been laid between one of the generating stations and a substation, rated at 115 000 kVA. The conductors are of 1·65 in.² cross-section, and the current rather over 500 A. per in.². The heating effect is recognized by making the summer loading lighter than the winter loading. This is not the first application of 66 kV. cables in Chicago. The use of the oil pressure cable allows of higher loading and thinner insulation, so that the cables are of no larger outside diameter than "solid" insulation cables of about half the kVA. capacity at the same pressure.

The "cost per ton of copper" plays only a minor part in the design of works of the "extra high pressure" class.

Summarizing the elements for calculation in more ordinary cases of l.t. distribution, for a load diagram of any shape—

(*a*) The mean square is the measure of the ratio of the copper losses to those which would pertain to the average current in the same conductor over the same period.

(*b*) The ratio of the mean square to the square of the maximum current is the load factor of the copper losses.

(*c*) The square root of the mean square—r.m.s. or form-factor—is the measure of the constant current which would give the same copper losses as those due to the varying current shown by the load diagram. The ratio of the r.m.s. value to the average constant current which would give the same ampere-hours product over the same period, is the divisor of the most economical current density (or the multiplier of the conductor section) calculated as most economical for the aforesaid average current.

(*d*) With the most economical current density for a varying current, the cost of the losses and of the capital charges due to the cost of the conductor calculated for the equivalent average constant current are both multiplied by the r.m.s. average ratio.

(*e*) The foregoing statements (*c*) and (*d*) are only true if the cost of the losses is constant, and is that taken for the calculation of economical density at 100 per cent load factor. The load factor of the losses is higher than that of the load current (excepting in the case of "rectangular" load diagrams), and the maximum loss rate at the peak of the load is proportional to the square of the maximum current; hence the average cost of the conductor losses will generally be higher than the cost taken for the 100 per cent load factor calculation, so that a lower current density than that indicated in (*c*) and (*d*) may be advisable. With pronounced peaks in the load diagram, the copper section may be entirely determined by the heating or voltage drop at the maximum load times.

Diversity Factor. This factor is the ratio of the total of a number of individual maximum loads to the aggregate maximum resultant of their superposition. If the individual maxima are simultaneous, the ratio is unity; the diversity factor is then 1·0 which is its minimum value. If the total of the individual maxima is, say, 110 (kilowatts, kilovolt-amperes, or amperes) and the aggregate maximum is 100, the diversity

factor is 1·10, and so on for any other ratio. Diversity in this connection means diversity in the times of occurrence of the maxima of the component loads; its import in distribution problems is to define the ratio of maximum load capacity of substations, mains, etc., needed to meet the demands of consumers, assuming that their individual maxima are known.

There is a relation between the load factors of individual loads and the possible diversity factor resulting from those loads. If any group of consumers (i.e. loads) have identical load factors, the maximum possible resultant diversity is the reciprocal of that load factor. That maximum can only result if

(*a*) the load factors are "period of use" factors; and
(*b*) the individual loads are equal and do not overlap in time.

For example, if four consumers each use full load for 6 hr. per day, and no more, their load factors are all 25 per cent. If each uses full load for a different six hours, none of them overlapping, the resultant load on a substation supplying them will be that of one consumer. If all four loads are equal, the sum of them will be four times the resultant, i.e. the diversity factor will be 4, the reciprocal of the individual load factors. If there were ten consumers each using full load for 2·4 hr. per day and no more, not overlapping, the individual load factors would be 10 per cent and the possible diversity factor 10.

In general, the lower the load factor of any group (or class) of consumers, the higher is the possible diversity factor of the resultant. This has the very important consequence that the capital cost per kilowatt of consumers' maximum demand tends to become a smaller proportion of the capital cost per kilowatt of the system demand, as the consumers' load factor becomes smaller. In what degree the diversity factor will approach its maximum value for any consumer group having a common load factor is only ascertainable if the conditions are such that the individual loads and the aggregate loads of the group can be measured over a period. But some general ideas can be formed. Since low load factor implies a short-time use of maximum load, the probability of coincident maxima of individual loads diminishes with, but is not a characteristic of, the load factor. The probability that the maxima of a group of consumers whose chief load is shop window lighting will coincide on some winter day is high, though the individual

load factor may be quite low. On the other hand, the probability that the maxima power loads of a group of factories will coincide is low, even though their individual load factors may be high compared with those of the shop windows, and that they all work during the same hours. The diversity among residential lighting loads is certain to be higher than that among shop window loads, though there may not be much difference between the load factors over a year. The diversity among different classes of loads has also to be considered. If any pair of loads of equal magnitude do not overlap in time, each of them only costs the fixed charges pertaining to half its maximum demand. If one had sufficient data about the uses made of different kinds of loads by different classes of consumer, the calculation of diversity factors would be an application of the calculus of probabilities.

Experience, and measurement, combined with knowledge of the particular circumstances of use, will generally permit of a working approximation. The tendency in the past has been to underrate the effects of diversity, but with the greater attention now given to the subject, and the accumulation of experience, there is a better appreciation of its importance.

Diversity implies a summation of loads. The load factor of a distributor as observed at a substation is a resultant of the load factor and diversity factor among the consumers it serves. The load factor of a substation is the resultant of the load factors and diversity factors of the distributors leaving the substation. The relation between the aggregate of the maxima loads on the several distributors and the maximum load on the substation defines the plant capacity needed in the substation in terms of the aggregate. Working back towards the generating station, diversity between the demands on the substations has the effect of improving the load factor of the system, and of diminishing the proportionate generating capacity needed to meet the demands of individual consumers.

It is clear that the mixture of loads of different classes, lighting public and private, industrial power, traction, cooking and heating, tends to increase the diversity factor, and to improve the load factor of a system as a whole, i.e. as viewed from the generating station or other source of supply.

An evaluation of all the factors treated of in this chapter is essential to a complete cost analysis of any existing distribution system. Carried out in detail for each subdivision of the system,

and applied to the relevant capital charges, costs of production, etc., it permits of the ascertainment of the cost of the energy delivered to and by that subdivision. It can be extended to find approximately the cost of supplying each class of load and each defined consumer group. That extension requires data from measurements which it is not at present usual to make, but which seem to be well worth while making in sufficient detail to afford reasonably approximate average figures.

The practical bearings of such analyses are (*a*) to detect opportunities for improving the economy of the system; (*b*) to afford data for the design of extensions either of capacity or of area of supply; and (*c*) to afford bases for the formation of tariffs applicable to consumers.

Some of these applications fall to be considered in other places.

CHAPTER XVI

TARIFFS

The subject of pricing supplies of electrical energy to consumers has been discussed from many points of view and over a long period. The economic principles of such price fixing are not different from those which govern the prices of other commodities and services; but the conditions of consumption affect the cost of the service to a greater degree than in most, because the capital and other overhead costs are larger and are more affected by those conditions.

In most countries there are legislative or administrative prescriptions limiting prices, with the object of protecting consumers from exploitation by the undertakers, justified in principle by the fact that the undertaker has been granted powers in excess of common law civil rights and has a more or less complete monopoly of supply to the public within the area covered by the grant of powers.

Here the common statutory provision, dating from the first Electric Lighting Act of 1882, is that no preference may be shown as between consumers in like circumstances, a provision which has important consequences. Another prescription of the same origin is that of a maximum price, fixed in the Act or Order granting the powers and liable to revision after certain procedure. There are also provisions affecting prices for bulk and special supplies in certain cases. It is needless to detail them: the undertakers subject to them know what they are.

The most important operative condition affecting the costs of supply is that the undertakers' plant, from generating station to the consumers' services, must be capable of carrying the maximum demand which may be made upon it at any time. Hence a part of the capital charges involved in supplying any consumer depends upon the maximum demand he makes at any time, independently of the duration of the demand, and of the total energy consumed by him in a given period. A tariff based upon the cost of supply must have an element of capital charges, though that element may not be formally expressed in the tariff. The service cable, meter,

etc., for each consumer entails capital and maintenance charges incurred exclusively for such consumer; but working backwards via the distributing mains, substations, h.t. feeders, etc., to the bus-bars at the generating station, the capital charges for each stage become divisible among an ever-increasing number of consumers. There are time differences in the maximum demands of consumers, so that beyond the individual services, and so forth, the correct apportionment of the total fixed charges upon the distributing plant depends upon the measure of simultaneity of the individual demands. The conduct of the whole number of consumers in respect to the time incidence of their maximum demands controls the average fixed charges cost per kilowatt of total maximum demands. The time distribution of the maxima is commonly expressed as the *diversity factor*. One result is that most of the distributing plant can be designed for less than the total of the consumers' maximum demands, as has been set out in detail in previous chapters.

To determine exactly what every consumer costs under the heading of fixed charges is impracticable.

To revert to common principles of price fixing, an essential condition for an undertaking to remain in a stably solvent condition is that the whole of the receipts shall at least equal the sum of the capital charges and of the working costs. How to make the price tariff do that, when the conduct of consumers —as well as their consumption—affects the costs, and to obtain all the business possible on remunerative terms, are the dominant problems of tariff planning. "Obtaining all the business possible" requires that the tariff shall attract consumers of all classes, which means, *inter alia*, that it should be readily understandable by consumers, and give them some idea of what the service they desire will cost them. It must be reasonably equitable as between different consumers, and it must look equitable, not necessarily the same thing.

The potential consumers of electrical energy in most areas may be divided into two main classes; (A) those who require a supply for commercial purposes, where the cost is a working expense of the business; and (B) those who require a supply for domestic purposes in their homes.

This division is not based on the use made of the supply of energy, but on the two objects in view: commercial gain on the one hand; home comfort and convenience on the other.

(a) **Commercial Tariffs.** The potential manufacturing consumer considers the terms offered to him from the same point of view as he considers tenders for the supply of raw material, he compares the terms with those upon which he can obtain equivalent service, i.e. competitively. The possible competitive agency may be a power plant of his own; he will make or obtain estimates of the capital cost and running expenses; he will (or should) take into account the occupation of site or buildings, the incidentals of having a power production department added to the manufacturing departments, and to the sinking of capital which might otherwise be "working capital" of his business.

In most areas of supply in this country there are or may be factories and the like large enough to make the setting up of a private plant practicable. To obtain such a consumers' business the supply undertaking is virtually in competition with a private plant proposition. That will fix the maximum terms upon which the business can be secured. But as it is the prospective consumer who makes the comparison and puts his own value upon some of the items on both sides, the undertaker cannot usually know what terms will prevail against a private plant proposition. The "no-preference" principle comes into operation. If one factory owner gets certain terms, those terms cannot be refused to others "in the like circumstances." The "circumstance" that the first is in a position—or professes to be—to put in his own plant is not one which a Court would be likely to consider as a good reason for refusing similar terms to others not so well placed.

A large manufacturer being accustomed to analyse costs and the incidence of overhead expenses, will probably appreciate the propriety of making load factor an element in the tariff; so that a tariff for manufacturing uses with a load factor or maximum demand element will be acceptable, and if soundly based will be remunerative even for smaller manufacturers.

It would certainly be an unlike circumstance that a large consumer pays for kilowatt-hours at high-tension, while another pays for kilowatt-hours at the general distribution voltage; or that one has an "off peak" load and another no time restrictions.

For some industrial processes electrical energy comes into direct competition with other agents, coal, gas, oil, etc.: especially

is this the case where heating processes are concerned. On the bare cost of gross thermal capacity, electrical energy from steam-driven stations cannot compete with such fuels. The advantages of adaptability, efficiency, cleanliness, close and automatic regulation of temperature, and so forth, have money values which are not easy to assess without actual trial, but have been proved to be substantial in many cases. Where an attractive load is in prospect there may be a temptation to offer terms which approximate to the "therms" costs of competitive agents. That temptation should be resisted. Tariffs should not fall below costs when all the conditions of service are fairly assessed. In the long run it is not to any one's interest that a less economical agent should be preferred over a more economical one. Any consumer charged below cost is, in effect, subsidized by others. That applies all round, not only to industrial consumers.

Large hotels, theatres, kinemas, and the like may also be in position to install their own plant; but the conditions of consumption are sufficiently diverse from those of factories as to justify other terms, unless the load factor or maximum demand rate takes adequate care of such differences.

The shopkeeper also has to look at the terms offered from the point of view of business costs. In his case the cost becomes of the nature of an overhead charge, not of a "per unit" addition to the cost of the goods he sells. There is no effective competitive agent as regards internal shop lighting; the shopkeeper must have it. (That is probably also true as regards external shop lighting.) But he is a keen bargainer; he would not be a successful shopkeeper otherwise. He is resentful of rates higher than those charged to other classes of consumers. As a class shopkeepers often wield a good deal of influence with local authorities, and make use of it. Shop lighting has a poor load factor, its maximum demand often coincides with the annual peak—sometimes it makes the annual peak; and it should be charged accordingly. The maximum demand system is logically suitable, but it is not popular. But some form of special tariff is necessary for the business to be remunerative to the undertaker. A fixed charge per 100 watts of connected lighting load has considerable vogue. If that charge is properly assessed, the consumption charge per kilowatt-hour can be very moderate; it is an advantage to make it the same as the kilowatt-hour rate in a current two-part tariff for domestic

consumers. That should encourage electric heating and cooking (if the shop runs a canteen or restaurant) as well as the use of small motors, fans, etc. Special "off-peak" rates for display lighting after closing times are attractive to some trades.

(b) **Domestic or Residential Tariffs.** There has been more controversy and more diverse practice about charges for domestic consumers than about any other branch of electricity supply commercial policy. The number of tariffs and the range of rates are bewildering. Potentially, domestic consumption is of enormous magnitude. Actually the consumption is limited by the consumers' means. A potent restrictive operator is the cost of wiring and appliances. The less well-to-do, i.e. the majority, are not able to make relatively large cash payments for such things. These people have to be assisted by hiring and hire purchase schemes which substitute periodical instalments for lump sum payments. Such schemes are strictly outside the scope of this work. They involve the undertakers in capital costs beyond those pertaining to supply. The terms should be such as to cover the resulting charges. It may be said that any adverse balance on this account may be regarded as a cost of getting the business, and charged as a sort of "advertising cost" against the profits from supply. For what it is worth the author's opinion is that the argument is of doubtful validity. As a practice of honest accountancy the costs of the assisted installation business should be kept and shown separately. The matter is not of small relative importance. In many cases the capital involved in a consumer's installation is of much the same order as the capital employed in supplying him. Properly priced and accounted for, instalment payment facility is a perfectly proper way of obtaining the business; the initial capital ought to be redeemed out of the hire charges within a few years.

Generally, in this country, every consumer within easy reach of the mains is entitled to require a supply and the undertaker has to give it. The undertaker cannot charge more than the statutory flat rate, but may charge for a minimum of (usually) 20 kWh. per quarter at that rate, even if nothing has been consumed. Lighting is the primary requirement. It has convenience and amenity values over lighting by any other agent that outweigh considerations of cost; though nowadays electric lighting is as cheap or cheaper than other agents.

Domestic lighting has a poor load factor: many domestic

consumers are not remunerative as long as they are only lighting consumers, even at the statutory maximum price. (This is not to say that *as a class* domestic lighting consumers are not remunerative at the statutory rates or even much lower rates, but there are always some with load factors of the order of 5 per cent or less who cost more to supply than the flat rate or the minimum charge.)

The poor load factor of lighting is partly compensated by a considerable diversity factor among the consumers. Experience shows that the collective load factor of large groups is of the order of 10 to 12 per cent. A flat rate calculated to be remunerative for a consumption with that load factor will usually be below the statutory maximum rate, and will attract a good many consumers. But it is not economically satisfactory to get no more than a 12 per cent "utilization factor" of the undertaker's plant. Electric heating, cooking, water heating and similar services, have considerable amenity advantages for domestic use, but at the lighting flat rate are more expensive than other agents, and beyond the means of the majority. They constitute a far larger potential consumption than lighting, a strong incentive to the undertakers to devise tariffs which will secure such loads and be within the means of the consumers.

Cooking, water heating, and electric fires produce much larger loads per consumer than lighting. But the diversity factor is also much larger. The aggregate load factor of a number of consumers with cooking, etc., appliances is considerably higher than with lighting only. Also the time incidence is such as to even up the daily load curve; the peaks do not coincide with those of other classes of loads, not even with that of domestic lighting. Hence a lower overall rate than the lighting is at once commercially justifiable and necessary to attract the business,

The maximum demand tariff is not suitable for domestic consumers. It does not take account of the diversity among consumers, or of the time incidence of the maxima loads, and it tends to restrict the connected load. When it was first introduced, practically all loads were lighting loads; the diversity factor was not high, most people made their maximum demands at about the same time. It did rough justice between consumers, and had a considerable measure of success in encouraging more generous use of the service. Consumers who

understood the system and could control their loading conditions found it advantageous. But understanding was not general, and the average consumer did not like it. There was the cost of the demand indicators, the additional readings and calculations for billing, additional questioning of accounts by consumers; so the system has dropped out of use for domestic consumers generally. But it remains for large scale use, such as industrial and bulk supply, as the nearest approach to an equitable system for them.

The principle that any tariff should include an element covering the capital and fixed charges involved in serving each consumer is difficult to apply in the case of domestic consumers. But as it is desirable that the charge per kilowatt-hour consumed should be low in order that other than lighting appliances should be installed and freely used, it is necessary to have some way of securing the fixed charges; the real problem of domestic tariffs is to find the best way.

Most of the methods in use are some form of *two-part* tariff. A fixed amount per quarter is charged and the consumption is charged at so much per kilowatt-hour; the sum of the two is the amount of the consumer's bill for the quarter. A very widely used basis for the fixed charge is the rateable value of the house. Another is the floor area, or the number of the living rooms, or some other dimensional factor. Sometimes the charge is assessed upon a basis not stated after inspection; presumably rough justice is aimed at. One basis which has been suggested —but not much used, if at all—is the connected lighting load.

None of these is a rational method of assessing what each consumer costs the undertaker in fixed charges; all are open to criticism on the ground of fairness as between consumers.

The rateable value basis is the favourite with local authority undertakers; a dimensional basis with companies.

The consumers' attitude to those fixed charges is that in a sense they mean paying for nothing, but in practice they are not very much concerned about that if (*a*) the fixed charge is moderate, and (*b*) the total cost of the service will be within their means.

A consumer who has only lighting will possibly see that a two-part tariff offered him will increase his quarterly bill as compared with the flat rate at which he has hitherto been charged. He will not change over unless he wishes, or is persuaded, to adopt cooking, heating, etc.

Consumers of the weekly wage-earning class dislike periodical bills. They are extensively catered for by prepayment meters. Great ingenuity has been spent on the design of prepayment meters which collect fixed charges, including those for wiring, hire of appliances, and so on. Such meters are necessarily more expensive in first and maintenance cost than plain kilowatt-hour meters with a prepayment attachment, but they are in successful use. It may be taken as proved that for the wage-earning classes prepayment meters are essential.

Two-part tariffs have succeeded in obtaining large accretions of domestic load, despite the more or less valid objections to the basis of the fixed charge element. The degree of success depends upon the facilities offered for acquiring appliances, and the quality of "salesmanship" employed in persuading people to try new things. It is interesting to notice that where success is most marked, the load curves of the undertakings have been altered for the better. In some areas the peak load has shifted to Sunday mornings, dinner-cooking time.

Less common in this country is the *block* tariff. For the first block (kilowatt-hours per quarter or other period), the kilowatt-hour rate is a maximum, decreasing with successive blocks to some minimum. The size of the block at each rate is fixed for each consumer, on some estimate of his maximum demand. It is really a kind of maximum demand system, but based upon an estimate instead of an actual demand, and not open to some of the objections to the use of the actual demand; e.g. the payment is not put up by an occasional high load, or by additions to the appliances installed.

In principle the block system is far from being equitable, but it seems that it can be made reasonably fair in practice by taking account of the load characteristics pertaining to distinct classes of consumers. Some awkward questions about "preference" seems possible under the system. The system is rather widely used for domestic consumers in the United States. The North Eastern Electric Supply in this country is reported to have introduced it in place of a two-part tariff for similar classes. Usually the kilowatt-hour rate for the first block is the lighting flat rate. The minimum rate corresponds to the consumption rate for a two-part tariff. The fixed charges element of the costs is met by the excess over that rate of the earlier block rates. Evidently the extent to which the tariff

meets the requirements depends upon the accuracy with which the demand costs have been forecast and the estimate embodied in the tariff.

All these methods of assessing and collecting the fixed charges are open to the criticism that they are arbitrary and cannot do more than "rough justice" as between consumers. On first principles, any tariff whatever that has the effect of charging some consumers less than they cost, and charging others more than is needed to make them remunerative to the undertakers, is detrimental to the business. It encourages the less and discourages the more remunerative classes. But it is impracticable to determine what each domestic consumer costs. It is practicable to make an approximate estimate in many cases of large consumers of the commercial classes. Getting the business of such consumers on remunerative terms often depends upon the accuracy of such estimates, as has been pointed out above.

It is possible to make a fairly good determination of what a particular group of consumers has cost in a given period, by measurements of input to the substations and distribution mains serving the group, including the load curves of the group, and comparing the measurements with the aggregate meter readings of the group. Additions to the capital charges of distribution for meter readings, clerical work, etc., are readily made. From such measurements it is possible to work out flat rates which are correct for different classes of consumers, i.e. they will in the aggregate cover all the costs and charges applicable to each class.

It seems to the author that this is really the best kind of tariff for domestic consumers, and is applicable to other classes whose load characteristics have been ascertained. The difficulty of application lies in determining how to define the class to which each individual belongs. On the whole the proportions of connected load of different kinds seems the most practicable and the most equitable way. Reasonable ratios per kilowatt of lighting, cooking, fires, and water-heating with and without storage, seem to be attainable. If any kind of limitation of load at peak periods is adopted, that can be allowed for in the rate. This procedure amounts to considering each "class" as one large consumer and fixing a rate which takes account of the aggregate load characteristics as if they pertain to that entity. Obviously it gives an average; so some individuals of the class

may be over-charged and some under-charged. But that criticism applies to every method; most forcibly to a uniform flat rate tariff.

The system of classified flat rates would obviate the objections to fixed charge payments, simplifycl erical accounting labour, and simplify also prepayment meters. It does not secure perfect equity as between individual consumers, but can be made equitable as between different classes, which is as near as is practicable for the small consumers' classes. It is not known that it has been tried in its entirety anywhere. But the necessary measurements have been made and the results published in certain cases, as noticed in a preceding chapter. Apparently some applications are being made, but rather by way of classified block tariffs than by classified flat rates. The overall results are much the same; the equity as between different consumers depends upon the accuracy of the assessment of the demand element in the application of the particular block scale to each.

An attractive idea is a uniform flat rate for all domestic consumers. That is being tried by a few undertakings and seems to have been successful, attractive to consumers and remunerative to the undertakings.

Such a tariff can make no claim to equity, but may not stray far from it if all consumers have nearly similar load characteristics. Obviously it would so stray if applied to, say, city offices on the one hand with little more than lighting load, and residences with cooking, water-heating, fires, and such-like equipment on the other. It is possible that in this country we shall come to uniform flat rates for domestic consumers. They can hardly be uniform in populous and in thin areas.

It has often been said that equity in supply tariffs is not of importance. Doubtless when a tariff, however based, is so low that a generous consumption results in charges well within the means of all domestic consumers, relative equity ceases to interest them. But the commercial class may well complain if they think that they are being over-charged for the benefit of residential consumers.

The measurement of loads, inputs, and outputs at each stage of a distribution system is necessary to keep watch over its detailed efficiency, and to show where changes are necessary to meet changed conditions. It should be standard practice— too much neglected hitherto—and afford a proper basis for

comparison with the sales revenue obtained from each portion
of the area ; and for a classified or other tariff designed to put
costs where they belong. That is the practical commercial
meaning of "equity"; it cannot be lightly ignored.

The commonly used criterion of "generating costs per unit
sold" puts the cost of distribution losses to the account of
generation—or of "energy purchased," which is defective
accountancy. Unless there is accurate knowledge of *all* dis-
tribution costs and charges, and of their locations, an essential
element for the guidance of commercial policy is missing.
For example, to offer close terms for a specially large supply
which involves additional mains, substations and the rest,
requires that knowledge in order to make a reasonable forecast
of the cost of giving the supply. It is equally important (and
often more difficult) to forecast the results of promoting
cooking, etc., loads in an area which has had a predominantly
lighting load, as regards the additions to the distributing
plant.

The establishment of the C.E.B. Grid is altering the basic
term of supply costs from "per kilowatt-hour at the station
bus-bars" to "per kilowatt-hour at the C.E.B. delivery point";
and as that cost is standardized for each area in a demand and
consumption tariff it gives a more stable basis. Consequently
distribution costs become relatively more important in the
framing of tariffs, and there is no longer the necessity to con-
sider the cost of providing new generating capacity to meet
new loads. Instead of that one has to take into account the
known rate per kilowatt of maximum demand in the area
tariff in relation to the estimated change in the load character-
istics of the system. Extensions of a "selected" station do not
involve added capital charges to be met solely from the con-
sumers of the undertaking owning the station. This gets rid
of the aforetime embarrassing "saw-tooth" curve of capital
charges per kilowatt-hour, rising at each increase of plant,
falling as the output grew up to the increment.

The tariff for supplies to tramways, trolley-bus lines, rail-
ways, etc., is often a subject of prolonged negotiation, even of
controversy, between the electrical and the transport commit-
tees of municipalities.

Where the transport undertaking is in a position to set up its
own station, the costs of supply from that station provide a
datum for the maximum tariff acceptable to the purchaser.

There is likely to be some difference in the estimates made by the two parties. If they are really independent, probably each party keeps its estimate details to itself. In this country the question whether a new station (or an extension to one existing) should be set up by the transport authority, is ultimately decided by the Electricity Commissioners who are in a good position to evaluate the estimates and arguments of the two parties, from the point of view of the general public interest.

But the minimum terms which a supply undertaking can safely offer to a transport undertaking are not dependent upon the circumstance that the latter can alternatively set up its own plant.

The technical peculiarity of a transport load is that it must have a separate terminal distribution system, the capital and working costs of which are its own affairs, so to speak. Hence the transport undertaking as a customer compares with any other taking a supply in the form, three-phase, high-tension, etc., at which the general distributing system connects into the transport system. The cost up to that point is the true comparative basis for an offer of traction energy. In British practice, the arrangements between supply and traction undertakings offer all possible variations, from the supply being delivered at station bus-bars, to delivery at the low-tension traction feeders. There are similar (but not always corresponding) variations in the ownership and responsibility for maintenance of high-tension feeders, low-tension feeders, substation equipment and the like. There are some municipal examples of the whole traction electrical equipment, including the trolley wires and poles, being provided and maintained by the supply department. Necessarily the different rates per kilowatt-hour charged to the transport undertakings under these widely varying conditions cannot be directly compared. A kilowatt-hour delivered to a traction feeder at 600 volts is not really "the same article" as a kilowatt-hour delivered at 6·6, 11, or 33 kV. at either the input or the output end of h.t. feeders. Nor is it the "same article" as the kilowatt-hour delivered at the meters of consumers on the general system.

In the author's experience, supply engineers are inclined to under-rate the diversity factor between a traction load and a general system load; they tend to regard the load factor of the traction supply by itself without making due allowance for that diversity, which is usually quite considerable.

That is nothing unusual; general experience is that the diversity factor of a new class of load with reference to existing classes is larger than was estimated in advance. Even the internal diversity of a new class is often higher than was expected. An outstanding example is cooking load, which in general has an internal diversity factor approaching ten.

Street lighting is a load the characteristics of which are defined by the lighting hours schedule specified by the highway authorities. It is usually an addition to the winter peak load and involves full costs per kilowatt as the demand element in the tariff, but as it has a high annual load factor the overall charge per kilowatt-hour can be low. Some part of the distributing system has to be special; generally the supply undertaking has to provide this. The provision of posts, lanterns, lamps, and their maintenance, may be wholly or partly in the charge of the supplier; these items have to be added to the costs and charges of the energy consumed. The rates cannot be directly compared with the rates charged to ordinary consumers.

Enough has been said to indicate how rates for different classes of consumption should be calculated in order to meet the fundamental principle that they should cover the specific costs referred to each class, with such margin as prudence dictates. It is assumed that some "profit" element is included in the fixed costs allocated to each class. The margin of prudence should depend upon an estimate with allowance for future changes in costs, and not more guesswork than is unavoidable. It may include an estimate of "what the traffic will bear," but departure from equity is likely to prove restrictive in obtaining business. A definite limitation to such additions is provided by statutory or regulative maximum prices—which may be below the cost of supplying some consumers—and also by the "no preference" principle.

It is hardly necessary to deal specifically with bulk supply tariffs. In this country these are generally prescribed by the relevant Acts and Orders, including the particular case of supply to and from the C.E.B. at "selected" stations. It will serve no useful purpose to discuss here the questions which have arisen between the C.E.B. and the owners of selected stations. These are subjects rather outside the scope of distribution economics, although the rates set certain values which have to be taken into account in basic costs.

A form of bulk supply tariff foreign to British practice is that known as *au forfait*. This is a contract demand tariff, so much per annum for a demand of up to so many thousand kilowatts. It is used for supply from hydraulic power sources where the running costs are unimportant compared to the fixed charges; mainly capital charges, including those on transmission lines. The purchaser has to pay the contract price irrespective of consumption. If his demand exceeds the contracted load, he has to pay for the excess at some kilowatt-hour rate. Frequently he has no right to exceed the contracted demand, or may only exceed it by special arrangement.

Every case of this kind has its special features. In some countries there are legislative provisions governing the tariffs, which may be embodied in the terms of the concession of rights to utilize the water power. Outside such provisions the determination of the rate per kilowatt which will pay the operator of the power source is a fairly simple matter if the extent of the market is known. To estimate what that extent will be, and what tariff the market will be able to bear, may involve some intelligent guess-work about future conditions.

CHAPTER XVII

SUMMARY

In the foregoing chapters the author has tried to show—

(1) Given certain data of prices, costs of installation, money interest rates, lives of the elements of a distributing lay-out; it is possible to design a most economical lay-out to suit a given area and distribution of loads having given load characteristics.

(2) In an existing system, it is possible by practicable methods to analyse the costs and charges pertaining to the various subdivisions of the system, in such a form as to show: (*a*) whether and how the overall economy of the system can be improved; and (*b*) what are the actual costs involved in supplying particular areas and particular classes of consumers in such areas.

(3) In a prospective system (which includes extensions of an existing system, or provision for increased loads in some or all parts thereof), there is an element of speculation as to the magnitudes, rates of growth, and load characteristics. Intelligent consideration of available statistics of comparable districts will give some guidance for such speculative estimates. In any case calculations can be made for assumed definite conditions, from which some limiting values and the variation of costs on either side of the most economical lay-out for given loads, or the most economical loading for a given lay-out, can be calculated. The order of accuracy of such calculations is not the same for all parts of the equipment, but is fortunately highest for such elements as mains which cannot be readily altered in future. Hence the variables for trial calculations can include various rates of growth of load; and therefore the period of time for which it is economical to make provision.

(4) The calculations aforesaid will give the aggregate costs (*a*) per mile at each operating voltage; (*b*) after each step of voltage transformation; (*c*) for each assumed load curve on the particular part of the system; (*d*) therefore for any specific load of which the load characteristics are known or calculable. Limits for variations in the load characteristics can be calculated.

(5) The most difficult element to forecast is the combined effect of individual consumers' loads upon the load characteristics of sections of the system. The diversity factor effect is particularly important. The diversity factor of a section as a

general rule is some inverse function of the average load factor of the consumers served by it. In the limit it is precisely the reciprocal of that load factor; a limit reached when all consumers have identical maximum loads and such load-time curves that the aggregate load is a constant. A rare condition, but the appropriate lay-out to meet it is a useful datum.

(6) Applying (4) and (5), a useful datum can be obtained for any proposed lay-out by calculating its economical loading at 100 per cent load factor; that defines the most generous provision of cables, etc., for the assumed load. The smallest practicable provision for the same load is that at which the voltage variation reaches the permissible limit; or, the cable sections, and so forth, which will reach the permissible temperature at that load. The heating limit involves duration of the load, or, rather the load curve over a complete cycle. Whether the voltage drop or the heating sets a limit depends mainly upon the length of main carrying the said load. In the particular case of cable loading it is simple to obtain a standard most economical current density for 100 per cent load factor and to make a proper deduction for any lower load factor. These values may have to be modified for either or both voltage drop or heating; it may be more economical to compensate for voltage drop by some form of regulator than to use a larger cable. These considerations are set out in more detail in the appropriate chapters. The economical choice of transformer capacity is more complicated than that of cable capacity. Fortunately, any deviation from the estimates on which the original choice was based can be more cheaply met by changing the transformers than by alterations to cable sections.

(7) The most expensive portion of a distributing system consists of the mains delivering at the consumers' voltage. In them the limiting factor is almost always the permissible voltage variation, a product of current and distance, i.e. the aggregate of amperes × yards from substations to the most remote consumers. The load capacity of an existing l.t. network can be increased by feeding it at more frequent intervals with a transformer at each feed point. This and the extension of a h.t. feeder to the added transformers is usually less expensive than adding to the l.t. cables.

(8) Increase of load in an existing system may tend to overload the h.t. network. The provision of an extra-high-tension system feeding into the existing h.t. network at or near

load centres is often the most economical way of increasing the h.t. system capacity.

(9) In regard to tariffs, the calculations and measurements mentioned in paragraphs (1) to (6) provide data for the costs of supplying defined classes of loads, i.e. of consumers, therefore of what terms can be offered, or what tariffs formulated. When the load magnitudes and characteristics are known, such terms and tariffs can be arranged to match the overall costs with close approximation. A maximum demand tariff is suitable for large industrial consumers and will not be unacceptable to them in general. For the more detailed formulation the last preceding chapter should be consulted. There is a large literature on the subject for any one who wishes to go more deeply into it. It is not sound, in the author's opinion, to have any tariff which charges less than the overall costs of supplying any class of consumers. It is inevitable, however, under British laws that some (generally small) consumers may be undercharged. It is good business to persuade such consumers to become larger ones.

The fixing of tariffs is not an exact science. As in most other businesses there is a range of prices which yields about the same net revenue. But the electricity supply business differs from most in that there are certain limitations imposed by the powers that be; and that there are competitive agents for considerable fields of utilization.

There is also the well-founded idea that it is in the public interest that the supply should be available at prices within the means of all potential consumers; that the business should be considered as a public service; and that the incentive of profit-making should be directed to secure large output at moderate profit, rather than the converse.

The author abstains from entering upon the thorny controversy about the relative merits of public authority and private enterprise in securing the desired results. That has been dealt with in other quarters and remains for decision on grounds by no means entirely economic in the specialized sense in which matters have been discussed in this book.

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